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THE

PHYSICAL REVIEW.

ATOMIC CONSTANTS AND DIMENSIONAL INVARIANTS

By A. C. Lunn.

Synopsis.

Dimensionally Invariant Products of Universal Physical Constants.—The number of experimentally determined universal constants is well known to be greater than the number of the fundamental independent units of measurement. The excess quantities should be expressible in terms of the others, that is, there must exist a number of dimensional invariants equal to this excess. Invariants which have a theoretical basis serve to check experimental values, and others also may be useful in suggesting numerical relations not otherwise suspected. As an illustration, this paper lists seven quantities which are dimensioned in length, time and mass only: e, the ratio of the electronic charge to the root of the dielectric constant; m, the electronic mass; $b = k^4/4^6\pi^6 a$, where k is the Boltzmann constant and a the coefficient of black radiation density; $f = h/2\pi$; n, the Rydberg wave-number; c, the velocity of light; and γ, the gravitation constant. In terms of these there must be four independent invariants, for instance: $c^3f^3/b = P$; $e^2/cf = S$; $\gamma m^2/e^2 = G$; $ne^2/mc^2 = E$. There are also many other relations, such as: $b/e^6 = L = 1/PS^2$; $\gamma^3m^6/b = G^3/L$; $mc/fn = S/E$; and $\gamma mn/c^2 = GE$. The only theoretical relations now known are: $P = 2^{12}\pi^8/15$, from the Planck spectrum distribution theory, and $4\pi E = S^2$, from the Bohr spectrum theory; both agree with experiment within 0.2 per cent. If in accordance with the Lewis-Adams suggestion we put L equal to 1 instead of 0.995, then Millikan's value of e must be too high by 0.5 per cent. Since S has the value 0.00730, $1 + S + m = 1.0078$, the atomic weight of H, and $4 + 2m = 4.0011$, the atomic weight of He; therefore S may equal the "packing effect." Various numerical formulæ for S are given, such as $3^3/5^3\pi^2$. If M_1 and M_2 are the ratios of the atomic masses of H and He, respectively, to that of an electron, $S^2M_1 = \pi/2^5$, and $S^2M_2 = 5^2/2^4$, within 0.1 per cent. Other similar numerical coincidences which may or may not have theoretical significance are given, for instance: $S/EM_1 = 2^7$; $E^6/GS = 1/2$; $G = 3(E/M_1)^4(1 - 2S)$.

IT has been recognized for some time that the number of universal quantities revealed by physical measurements, but not yet utilized for the practical reduction of the system of arbitrary units, is more than sufficient to complete that reduction so as to give for every sort of measurement a kind of natural unit. Since in most cases the measurements are not yet accurate enough for putting this reduction into effect, it is convenient to retain consideration of these quantities as having dimensions corre-

sponding to the scheme of units as it stands before the reduction is made. The redundancy means that the excess quantities should be expressible in terms of the others, and several relations of this kind have been proposed. Equivalently, there must exist a set of independent dimensionless expressions, or "dimensional invariants," expressible entirely in terms of the quantities of the list and equal in number to the excess.

Because of the differences in precision of different types of measurements it has been customary to adopt as basis a set of the better determined values, equal in number to the arbitrary or primitive units in the dimensional scheme, and for comparison with observed data to express the others in terms of these so far as suitable algebraic relations are at hand from theoretical sources.[1] There is some advantage, however, in avoiding restriction to such a preferential solution through the direct computation of a fundamental set of invariants. The influence of errors of observation in all the quantities concerned can be shown by simultaneous consideration of all the corresponding differential corrections, and a kind of test of the completeness of a theory is found in the number of independent invariants whose theoretical and observed values are in practical agreement. The intent of this paper is to examine the present state of theory from such a point of view, and to compare data from various experimental sources in connection with invariants which have had theoretical values deduced. There are also proposed, for enough further invariants to complete a fundamental list, certain tentative values so accurate as to suggest the possibility of deduction from a theory, and such as to give in particular, simple formulas for the action constant and constant of gravitation. Measured quantities are denoted by small letters and invariants by capitals.

The quantities to be included are the charge e and mass m of the electron, the bielectric coefficient of a vacuum ϵ; the Boltzmann constant k and the coefficient a of black radiation density; the Planck constant h, Rydberg wave-number n, velocity of light in vacuum c, and constant of gravitation γ. There are nine of these, to be considered as dimensioned in five primitive units, say the familiar three of mechanics, together with temperature and one electrical unit. Since the temperature unit affects only k and a, and the electrical unit only e and ϵ, it will be sufficient to consider only the ratios k^4/a and $e/\epsilon^{\frac{1}{2}}$, which are dimensioned in L-T-M only. The latter ratio may as well be denoted by e simply, and it proves convenient to use $k^4/a(4\pi)^4 = b$ say, also in place of h to use $h/2\pi = f$ say,

[1] Compare for example the computations in Sommerfeld: Atombau und Spektrallinien, ed. 2, pp. 367–370; Flamm, Phys. Zeitsch., 18, 515–521, 1917; Allen, Proc. Phys. Soc. London, 27, 425–431, 1915; Millikan, PHYS. REV. (2), II., 109–143, and Phil. Mag. (6), XXXIV., 1–30, 1917; Birge, PHYS. REV., (2), XIV., 361–368, 1919.

the natural unit of angular momentum. There are then the seven quantities e, m, b, f, n, c, γ, dimensioned in L-T-M simply, and therefore having a fundamental system of four invariants.

Together with these four there needs to be considered any relevant ratio of two quantities of the same kind which are not interpreted in terms of each other by a counting process. So far as is known all charges are integral multiples of e, and the b, γ, c, and h are unique, though there seems to be some possibility that the slopes of the photo-electric lines for different elements may prove to involve slightly different individual factors, as well as a universal h, when a precision is reached comparable with that of the various wave-numbers of metallic series. So far as is satisfactorily made out the latter are expressible in terms of the universal n and certain functions of integers together with factors depending on mass-ratios. Moreover, there seems to be increasing probability that all pure atomic species will prove to have atomic weights integrally related to hydrogen or helium or some compromise of these, and the suggestion doubtless remains acceptable, though not yet definitely successful, that the departure of the helium-hydrogen ratio from the integral value may be explainable by an overlapping of fields or "packing effect," or something similar. This would leave the single mass-ratio of hydrogen atom to electron to be explained, but since there are no isotopes of either hydrogen or helium known it seems best to consider both of these mass-ratios, $m_1/m = M_1$ for hydrogen and $m_2/m = M_2$ for helium, as requiring interpretation. A theory of matter can hardly be considered satisfactory until it reaches some interpretation of these ratios, rather than treating them as primary data, but for the present purpose it is suitable simply to annex them to the list of invariants to be studied.

The list of quantities might be taken to include also the gas constant and Faraday constant per mole, but since this involves the use of an additional unit, the chemical mole, there would be only one more invariant, which can be taken as kcF/Re of value unity. Further, the specific charge η can be treated either as abbreviation for e/cm or as a distinct quantity associated then with the invariant $cm\eta/e$, also unity. The number of outstanding invariants required is the same as before.

The algebraic work of determining invariants as products of powers need not be detailed. Since it is likely that the more direct results of theory will be concerned with relatively simple invariants there is convenience in arranging according to number of factors. Examination shows that in the list of seven quantities named there is only one pair, (b, e), which yields a two-letter invariant, only four triples, (c, f, b), (e, c, f), (γ, m, e), (b, γ, m), giving three-letter invariants, and that between

these five there are two relations, so that to complete a fundamental system an invariant in four or more letters is needed, to be taken here as formed from (e, c, m, n). The following notation will be used and the identities are indicated.

$$b/e^6 = L,\ c^3f^3/b = P,\ e^2/cf = S,\ \gamma m^2/e^2 = G,$$
$$ne^2/mc^2 = E;\ \gamma^3m^6/b = G^3/L,\ PLS^3 = 1. \tag{1}$$

The L occurs in the paper of Lewis and Adams,[1] the S is identical with Sommerfeld's α and Allen's q (loc. cit.), the G is a ratio of gravitational and electrostatic forces, and the E can be read as ratio of two energies, the electrostatic potential energy of two elementary charges at a distance of one Rydberg wave-length and the energy corresponding in Einstein's sense to the mass of the electron. In general these four will be taken as fundamental, but a separate notation is taken for the P because of its occurrence in connection with Planck's radiation formula. The writer has also computed all further four-letter invariants and determined their values in terms of the four named. As examples may be noted $mc/fn = S/E$, which may be interpreted as ratio of two angular momenta, and $\gamma mn/c^2 = GE$, a ratio of energies per unit mass. Some of these other invariants may prove of interest in connection with future theories, but the present state of the problem can be described in terms of those already given. The values of all four would need to be known in order to give specific form and full effect to the vague but suggestive theory of Lewis and Adams concerning the ultimate rational units.

There are required then six independent relations for the determination of the four fundamental invariants and the two mass-ratios. Two such relations have been deduced from theories, one coming by integration from the Planck spectrum distribution formula, the other being Bohr's formula for the Rydberg constant. These relations in the present notation are

$$P = 2^{12}\pi^8/15, \tag{2}$$
$$4\pi E = S^3, \tag{3}$$

and their accordance with experimental data is known to be as accurate as the measurements used. Other equations already proposed and those to be added in this paper are to be understood simply as possibly deducible from some theory and by their form possibly suggestive of some features of such a theory, although in any case a relation between invariants represents an experimental fact with whatever precision it proves to have, even if its theoretical connections are unknown. Of

[1] PHYS. REV., (2), III., 92–102, 1914.

the latter type is the relation suggested by Lewis and Adams, namely

$$L = 1 \tag{4}$$

which is so close and simple as to have high probability.

The relations suggested by Allen are not included here, since they are not relations between invariants and depend for their particular form on the use of special units and the decimal scale of notation. A relation proposed by Haas,[1] equivalent in the present notation to $\log G = -2M_1^{\frac{1}{2}} \log \pi$, involves only invariants, but its plausibility is difficult to estimate, partly because a small percentage change in M_1 corresponds to a much larger percentage change in G, about fifty times as large, so that data now known are rather rough to test it. If really correct, however, it yields an accurate value of M_1 for that reason.

Initial test computations will be made with the following data: $n = 109737$, $c = 2.999 \times 10^{10}$, $\gamma = 6.675 \times 10^{-8}$, $e = 4.774 \times 10^{-10}$ $h = 6.545 \times 10^{-27}$, $F = 9649.4$, $\eta = 1.767 \times 10^{7}$, $ca/4 = 5.72 \times 10^{-5}$, giving $m = 9.009 \times 10^{-28}$, $a = 7.629 \times 10^{-15}$, $b = 1.178 \times 10^{-35}$, $f = 1.042 \times 10^{-27}$; and with $M_1 = 1845$, $M_2 = 7328$, corresponding to atomic weights 1.0077 and 4.0020, though the precise values and meaning of these ratios is one subject of discussion.

From these data come $P = 2.588 \times 10^{6}$, $S = 7.295 \times 10^{-3}$, $E = 3.087 \times 10^{-8}$, $G = 2.377 \times 10^{-43}$. Sommerfeld's value of S, computed directly from spectrum data, is 7.290×10^{-3}, but since this comes from the observed separation of components of fine structure having each some outstanding broadening and asymmetry the measurements are not accurate enough to distinguish this value from the other or from the hypothetical values to be mentioned. The values of $15\,P/2^{12}\pi^8$, $4\pi E/S^3$, and L, which would all be unity according to the equations cited, are respectively .9985, 9989, .9952. Evidently any one of these relations separately can be regarded as exact without stretching the range of uncertainty in the data, but the question is rather whether a single system of differential corrections of admissible magnitude could be applied to the data so as to satisfy all three relations at once. A least square solution with a tentative set of weights gave corrections to both e and η rather dubiously large, but since the assignment of weights to such varied data is precarious the dilemma involved is better shown as follows.

The Planck and Lewis-Adams equations, because of the identity connecting P, L, and S, determine a hypothetical value of S by

$$S_{PL}^3 = 15/2^{12}\pi^8, \quad S_{PL} = 7.281 \times 10^{-3}, \tag{5}$$

which by itself is clearly admissible; but the inclusion of the Bohr equa-

[1] Haas, Phys. Zeitschr., 19, 330, 1918.

tion gives for E a value

$$E_{PLB} = 15/2^{14}\pi^9 = 3.071 \times 10^{-8}, \quad (6)$$

whence $E/E_{PLB} = 1.005$. There seems to be no known reason for regarding the Rydberg constant as other than sensibly exact for the present purpose or the velocity of light as more uncertain than to the extent of a unit or so in the fourth figure, so that the implication of this last ratio is practically that the product of the measured values of e and η is ½ per cent. too high. Since the η has been measured in a variety of ways by a number of observers and checked by Sommerfeld's spectrum computation with such consistency as not to encourage a reduction of more than one part in 500 or so, the value assumed being probably already low, it appears that the validity of all three relations mentioned is likely to mean that Millikan's value of e is too high by nearly four or five parts per thousand, and that h requires an accompanying correction to keep reasonable accord with the photo-electric and x-ray data.

An uncertainty of that magnitude in measurements by the oil-drop method is doubtless conceivable and need not be ascribed solely or chiefly to instrumental errors unrecognized. There remains unexplained the unexpected lack of difference between solid particles and liquid drops of different viscosities,[1] which may point to some uncertainty as to what is the effective hydrodynamical radius in the application of Stokes' formula; or the hydrodynamical theory may be at fault in other ways, for instance because of the minute scale of the motions or because of the effect of compressibility of the medium, although no such corrections are known to be needed except the allowance for surface slip which was consistently made.

At present, however, the data make it somewhat doubtful that all three relations can be exactly valid, unless possibly by way of some slight change in meaning of some of the symbols. Because of its lack of a specific theoretical background that of Lewis and Adams seems most likely to be only a close approximation. The Planck theory is however fairly obscure in places and further refinements might change the spectrum formula slightly so as to alter the value of P without loss of accord with measurements on total radiation and spectrum distribution. The Bohr formula seems less likely to need amendment, not so much because its theory is altogether satisfactory as because of its wide range of contact with spectrum data, but in view of the influence of motion of the nucleus and the relativity corrections it seems not impossible that other factors may affect the determination of the ideal or asymptotic Rydberg constant to which the formula refers.

[1] Silvey, PHYS. REV. (2), VII., 87 and 106, 1916.

There seems to be no such difficulty in accepting any two of these relations without the third. The dilemma described can be relieved merely by supposing that (3) is not quite exact; conceivable substitutes for $L = 1$ nearer the computed value will occur in the following; while equations (3) and (4) without (2) leaves S undetermined and its elimination gives simply $4\pi EP = 1$, the computed value being 1.0038, not seriously different in view of the number of factors involved. To illustrate these possibilities it may be mentioned that Millikan's e and Duane's latest value for h,[1] 6.556×10^{-27} using the same e, tend to favor the Lewis-Adams relation rather than the Bohr, giving $S = 7.283 \times 10^{-3}$, curiously near to S_{PL}. Though the estimates of uncertainty given by these observers are somewhat more than enough to cover the discrepancy the numbers as they stand would require in order to satisfy (3) a ½ per cent. reduction in η. But the Einstein quantum relation used in reducing the x-ray data may itself be in need of refinement by factors approximately unity but specific for particular substances; this would be only natural perhaps in view of the nucleus and relativity corrections that improved on Bohr's original equation. Moreover, a complete review of these problems might even find some uncertainty concerned with the Faraday constant, defined in connection with a heavy atom and used for such important purposes in connection with the lightest ones; and this not only because of the recognized difficulties in the way of verifying the Faraday law with the required precision over the whole range of atomic weights. The theory of electromagnetic mass has made familiar the notion of "mutual" mass, whose main descriptive content is concerned with the idea that the summational property of mass may cease to be valid for sufficiently fine subdivisions of matter. This suggests that the strict proportionality of weight, mass, and number of ions may be true only for aggregations of sufficient size to blur out differential effects due to characteristic differences between positive and negative electrical constituents. In such a case the ratio of inertias of individual ions might not agree with the ratio of the large scale chemical equivalents. Positive-ray analysis may reach precision enough to give this possibility a definitive test. At present the prospect is that any such correction would be unimportant for complete atoms.

Related to this is the reason for including the gravitation constant in the present list, namely the widely cherished conviction that gravitation should be interpretable as a residual electromagnetic effect. Some attempts at such a theory seem rather bizarre, and those involving arbitrary statistical assumptions are not likely to be satisfying, one aspect

[1] Proc. Nat. Acad. Sc., 7, 237–242, 1921.

of the difficulty being the very smallness of the residual required, as reflected in the minuteness of the invariant G; but now that fundamental differences between positive and negative electrification are experimentally real the outlook seems more hopeful. Since the idea is used here only as a hint, indication may be deferred as to how it can be considered to harmonize with Einstein's interpretation of the gravitational field.

There are many examples of successful formulas whose construction to fit empirical data antedated their deduction from theory; the Planck and Balmer-Rydberg formulas are examples. It may be not altogether idle, therefore, to search for further numerical coincidences and test them out in various combinations. Especially may be noted such relations as may be in general form suggested by tentative considerations of theory, but too vague to determine for instance certain numerical coefficients.

Aside from the gravitational formula, the kernel of the problem seems likely to be the determination of the invariant S, which is already fundamental in Sommerfeld's theory and will doubtless prove to be of much wider importance. A theoretical explanation of its meaning and value will probably mark the achievement of a satisfactory logical connection between the electron theory and the quantum theories. One suggestion concerned with this outlook appears with the following items, which may not be accidental; fuller developments and alternative interpretations are saved for a special paper.

First, the expression $1 + S$ has value 1.00729, uncertain hardly as much as two units of the last place; addition of .00055, the "atomic weight of an electron," gives 1.00784, agreeing with the atomic weight of hydrogen as closely as that can be considered known. Addition of two electrons, .00109, to even 4.00000, gives conceivable atomic weight of helium differing from Heuse's value 4.002 less than his estimate of uncertainty .0017. The suggestion thus emerges that S may give a kind of measure, to first approximation perhaps, of the "packing effect" or influence of mutual terms which determines the departure of the helium-hydrogen ratio from the even integer.

Second, the notion of "mutual" mass suggests trial of a quadratic function of the atomic number, so as to represent atomic weight as the sum of a term proportional to the number of constituent particles and a term proportional to the number of pairs of particles. The formula $w = 2N + SN^2$ does in fact give quite well the general trend of atomic weights, especially when reduced to the definitional scale $w = 16$ for $N = 8$.

Because of the identity any trial value of S determines a corresponding value of the product PL, and hence with Planck's value of P leads to a

substitute for (4); but the partitioning between these two factors is better treated as a separate problem, these two invariants being the only ones used that contain the radiation constant a while their product is not affected by it.

If equation (3) is valid the range of uncertainty as to the value of S is rather narrow, in fact sifting of the data with that equation assumed shows only a small chance that it lies outside the limits .007270 and .007310; if only the values of e, h, and c directly concerned in the definition of S be considered the range is not seriously larger, since the value of h in terms of e by the x-ray method has given such closely consistent results. It was noticed by Jeans[1] that the value of $16\pi^2 S$ is approximately unity, but this is only a rough guess, the computed value being 1.152. On the other hand the value proposed by Page[1] amounts to $S = 3/128\pi = .00746$, which is pretty high to be plausible.

One can hardly have much hope of obtaining a true theoretical value of such a constant by pure conjecture based on numerical coincidence merely, as is illustrated by the following list of numbers, any one a conceivable value of S: $32/45\pi^4 = .007300$, $7/960 = .007292$, $\pi/2^4 3^3 = .007272$, $11/480\pi = .007294$, $7/\pi^6 = .007281$, the fourth being practically the computed value and the last indistinguishable from S_{PL}. The possibility $S^3 = 3^3 2^{11}/5^6\pi^{14}$, $S = .007293$, is curious on account of the corresponding value $E = (24/25\pi^5)^3$. Some of these lead to sets of formulas on the whole fully as simple and plausible as those using the Lewis-Adams relation. But since such trials could be extended indefinitely, and since the coefficients in the later formulas to be proposed are similarly tentative it will be sufficient illustration of the possibilities to consider a single alternative. Since reasons have been given for supposing that the S_{PL} is likely to be too low, it is desirable to have an alternative toward the upper part of the admissible range. Accordingly is suggested for particular trial the new hypothetical value

$$S_H = 3^2/5^3\pi^2 = .007295, \tag{5a}$$

being practically the computed value, and with (3) giving

$$E_{HB} = 3^6/2^2 5^3 \pi^3 = 3.089 \times 10^{-8}, \tag{6a}$$

also practically as good a fit as the data require. Combination with (2) gives for (4) the substitute value $L = 5^{10}/2^{12}3^5\pi^2 = .9941$. But the possibility must be faced that the true value of S is no such simple thing at all. For example, if it measures some kind of coefficient of stability in atomic structures concerned with the mutual influence of overlapping

[1] Proc. Nat. Acad. Sc., 6, 115–122, 1921.

fields its physical meaning might be quite perspicuous but its numerical value determined only by some trancendental process. Similar remarks of course apply to guesses for other invariants, though such trials may lead to suggestions of promise. For instance, $(10 + \sqrt{5})^3 = 1832.0$, which may point to some trigonometric meaning for the apparent reciprocal atomic weight of the electron.

Since the known quantum phenomena are concerned not so much with the electron alone as with its relation to the various material atoms or nuclei, there is a prospect that the S may have something to do with the differences between positive and negative electricity. The chief known feature of these differences lies in the relatively small inertia of the universal negative electron as compared with the various positive nuclei. Hence there is likely to be some direct connection between S and a mass-ratio, though some uncertainty must be recognized as to what mass-ratio is most directly concerned. The distinction between the mass of an atom and that of its nucleus is at present rather trivial on account of errors in other data, but because of the $\frac{3}{4}$ per cent. departure of hydrogen from the integral value on the oxygen scale and the outstanding uncertainty as to the atomic weight of helium there remains the question as to what is the most directly natural theoretical unit of atomic weight. The notion of the proton as a universal constituent of all atoms suggests the hydrogen scale, the universality of the alpha particle in radioactivity suggests the helium scale, while the fact that the basic processes of measurement employ apparatus constructed largely of solid matter in bulk consisting of heavier atoms invites consideration of other possibilities. In connection with the following formulas it may be decided in particular cases whether M_1 and M_2 are to refer to the neutral atoms or to the nuclei, but the precision reached demands systematic distinction between the actual hydrogen-electron ratio M_1 and the reciprocal atomic weight of the electron, M say, on any chosen scale of atomic weights.

The order of magnitude of S, M_1, and M_2 suggests first consideration of the products SM_1 and SM_2, whose computed values are 13.46 and 53.46, the latter being close to $160/3 = 53.33$; but a form of relation more likely to be directly theoretical seems to be approached in the following way.

The electromagnetic radius of an electron on the hypothesis of a uniform volume distribution of charge is $4e^2/5c^2m = r$ say; the corresponding radii for hydrogen and helium nuclei are $r_1 = r/M$, and $r_2 = 4r/M_2$, these two being only slightly different because of the double charge of helium. An inquiry as to the meaning of these hypothetical "radii" in the light

of the theory of relativity led to examination of the geometric means $r_{01} = (rr_1)^{\frac{1}{2}}$ and $r_{02} = (rr_2)^{\frac{1}{2}}$, and it was noticed that the unit of angular momentum f was nearly the same as $4m_1cr_{01}$, the computed coefficients of 10^{-10} being 1.042 and 1.045. The factor 4 suggests examination of m_2cr_{02} which gives coefficient 1.041. Expressed in terms of the invariants these alternative trial relations become

$$SM_1 = 5/16 \quad , \quad SM_2^{\frac{1}{2}} = 5/8 \qquad (7a, 7b)$$

the computed ratios of left to right members being 1.0029 and .9993. Although the helium ratio gives here the better fit it is worth noting that if the hypothetical value S_{PL} be used the hydrogen relation becomes practically exact. The true relation may be some compromise deducible from some theory capable at the same time of explaining the departure of M_2/M_1 from the even 4. Though less simply interpretable an alternative possibility for the numerical coefficient appears from the fact that the computed value of S/EM_1 is 128.08, suggesting the first of the following, the second resulting from combination with (3),

$$S/EM_1 = 2^7 \quad , \quad SM_1^{\frac{1}{2}} = (\pi/32)^{\frac{1}{2}}, SM_2^{\frac{1}{2}} = (\pi/^8)^{\frac{1}{2}}, \qquad (8, 7c, 7d)$$

The left to right ratios of the last two are 1.0002 and .9966, the hydrogen formula here giving practically perfect fit. Since $8\pi/25 = 1.0053$ there is a curious possibility that both (7b) and (7c) might be valid if the atomic weight of helium were about 4.008; or (7c) and (7a) might correspond respectively to the true hydrogen and to some ideal unit of atomic weight. To decide between such closely agreeing alternatives requires greater certainty as to the values of the factors than is now to be had, the difference between (7a) and (7c) for instance being about $\frac{1}{3}$ per cent. Elimination of between (3) and (7c) gives

$$E^2M_1^3 = \pi/2^{19}, \qquad (9)$$

which shows test-ratio .9991 and does not involve the Planck constant. It is of course to be understood that any equation encountered as a result of combining other equations may prove to be valid independently of the validity of the particular relations through which it is initially arrived at. There may also be possibly useful suggestions obtained by writing the same equation in different forms. For instance equation (8) is equivalent to $fM_1 = cm\lambda/128$, where λ is the Rydberg wave-length $1/n$ and the left member is what the angular momentum of a proton would be if it revolved with the same speed and in the same size orbit as would give an electron the unit of angular momentum. There may be a hint here of some connection with the idea which associates the number

128 with a hypothetical decomposition of the hydrogen nucleus, 1/128 being .0078.[1]

A first suggestion for a gravitation formula is found in

$$2E^6/GS = 1, \tag{10}$$

the computed left member being .9975. Insertion of E from (3) gives

$$G = S^{17}/2^{11}\pi^6, \tag{11}$$

with left to right test-ratio .9963. This is a close fit in view of the high exponent of S, which also suggests however that a theoretical deduction would be likely to be rather indirect. Equation (3) and suitable relations of type (7) would make it possible to express in terms of S alone any function of E, S, and mass-ratios; and further forms would result by partial substitution of an assumed value for S.

There is also a certain artificiality in the use of G, which is expressed entirely in terms of electron constants. Since gravitation between electrons is purely hypothetical there is better chance of direct physical meaning, for instance in $G_1 = GM_1^2 = \gamma m_1^2/e^2$ or its helium mate, which with aid of an equation of type (7) would be according to (11) a multiple of S^{12}. It is also likely in this connection to be more fitting to use instead of the pure electron invariant E, similarly modified expressions like $E_1 = E/M_1 = ne\eta_1/c$ where $\eta_1 = e/cm_1$, or $E_{01} = E/M_1^{\frac{1}{2}} = ne^2/c^2(mm_1)^{\frac{1}{2}}$, which treats proton and electron symmetrically.

Another hint is to seek a formula not containing the action constant, say by elimination of S through some combination of (3) and a (7), so as to give G in terms of E and some M's.

A formula of this type, found in fact before (10), is

$$G = 3(E/M_1)^4 \equiv 3E_1^4 \tag{12a}$$

with test-ratio 1.012; this may be written $G_1 = 3E_{01}^4$. If the ideal M be used instead of M_1 the ratio is .9815. These errors are nearly enough equal and opposite to suggest the geometric mean, giving

$$G_1 = 3E^4/M^2, \tag{12b}$$

which tests .9967, which is at least as close as the data are known. The form of these equations seems prepossessing from the point of view of gravitation as a residual electrical effect. It is hardly surprising that the problem of the packing effects and the natural unit of atomic weight should be concerned here also, or that there should be initial uncertainty as to what mass-ratio can be taken as representative of the common

[1] Hinrich, Rev. Gén. d. Chim. pure et appl., 13, 351–354, 1910.

properties of matter in bulk. In view of the earlier suggestion of the possible bearing of the invariant S on that problem it is conceivable that the right members here need refinement by factors, for instance in the form of power series in S, with initial term unity, after the manner of Sommerfeld's improved formulas for series of the Balmer type. In fact (12*b*) is practically equivalent to a modification of (12*a*) of the form $G = 3(E/M_1)^4 (1 - 2S\cdots)$. Since $E/M_1 = 5r_1/4\lambda$ in the notation already used it is seen that these equations exhibit ratios of gravitational to electrical forces as proportional to the fourth powers of ratios of electrical to optical lengths. The possible meaning of any one equation naturally appears in somewhat different lights according to what quantities are used in writing it and which are thought of as fundamental. Other possibly significant combinations of equations of the types already given are readily obtained.

These various trial equations are proposed primarily as individual possibilities, but the basic problem is to obtain a complete set of equations, giving the full number of possible independent relations between the invariants simultaneously in harmony with experimental data. Since such a set has not hitherto been reached by theory or by conjecture it seems worth while to exhibit the following two samples as close approximations, the first being an extension of the original triple (2, 3, 4), the second using in effect a substitute for (4) while retaining the others.

In each case the program is to choose for trial a set of six independent hypothetical relations and by a least square adjustment find differential corrections to the initially assumed values of the seven dimensioned quantities and the two mass-ratios so that these relations are all satisfied. In general a suitable choice of weights for such a purpose would be more or less conjectural as well as the equations, and would influence the result importantly. In the present instance, however, the quantities m, c, F, equal in number as it happens to the number of primitive units, are by common judgment understood to be known with much greater precision than the others. Any reasonable set of weights then leads to a solution such as to leave the values of those quantities practically untouched, and in effect, to the number of decimal places sought, to determine the others in terms of them. The two illustrations may as well then be set forth in the latter sense.

Hypothesis I.—Equations (2, 3, 4) are all taken as valid, so S is determined by (5) then E by (3). The mass-ratio 1842.2 found from (7*a*) is assumed to be for the nucleus, giving 1843.2 for the H-atom, whose atomic weight is supposed to be $1 + S + .00055 = 1.00783$, hence $M = 1829.9$, and $\eta = FM = 1.7648 \times 10^7$. The value of E then yields $e = 4.756$

$\times 10^{-10}$, and that of S determines $h = 6.509 \times 10^{-27}$. The low value of the product $e\eta$ was earlier shown to be unavoidable under these assumptions and the partition seems reasonable. The value of $h/e^{4/3}$ is 1.753×10^{-14}, while Duane's value in effect has coefficient 1.757, the difference being a little less than ¼ per cent. and the error on the side toward the lower values that some other observers have obtained. The gravitation constant from (12*b*) is 6.683×10^{-4}.

Hypothesis II.—Equations (2, 3) are used with S and E determined by (5*a*, 6*a*). Then M_1, here also taken as for the nucleus, by (7*c*) is 1844.8, for the atom 1845.8. Atomic weight of hydrogen as in I gives $M = 1831.4$, so $\eta = 1.767 \times 10^7$, and $e = 4.778 \times 10^{-10}$, $h = 6.556 \times 10^{-10}$, $h/e^{4/3} = 1.755 \times 10^{-14}$, and then (10), used this time for the gravitation constant, gives $\gamma = 6.696 \times 10^{-8}$.

It seems thus quite possible that some of the equations here proposed foreshadow theoretical results of considerable directness, of a character as required for completion of the theory as it now stands.

THE UNIVERSITY OF CHICAGO,
February 19, 1922.

THE RATIO OF THE TWO ELEMENTARY CHARGES.

By F. W. Loomis.

Synopsis.

Since the oil drops in Millikan's experiments were exactly neutral, = a small integral number of electronic charges, to 1/1000 of an electron, and since they contained as many as 4×10^{14} protons and nearly the same number of electrons, it follows that *the magnitudes of the charges on protons and electrons are equal to one part in* 4×10^{17}.

IN recent years and particularly since the work of Aston it has become generally accepted that all matter is made up of protons and electrons. It is usually taken for granted that the charge on the electron is the negative of that on the proton, but the exactness of this relation is rarely discussed.

Some information can be obtained from positive rays. Aston[1] has found that the positively charged atom of hydrogen, *i.e.*, the proton, shows a deflection corresponding to $m/e = 1.008$ (in units of atomic weight and electronic charge). Since it is a pure element and its chemical atomic weight is 1.008, the magnitude of the charge is found to be equal to that of the electron within the accuracy of Aston's measurements, or 1/10 of 1 per cent. This accuracy is sufficient to establish that the number of protons in a "neutral" atom is equal to the number of electrons, including of course those in the nucleus. Hence it is possible that a more precise measurement of the ratio of the charges might be made by measuring the deflectability of the "neutral" atom; though there would be difficulties due to the confusion of the trace of the "neutral" particles with the darkening caused by the light from the discharge tube.

But much the most precise value of this ratio can be deduced from Millikan's oil-drop experiments.[2] Millikan measured the net charge on oil drops charged in several ways with from 1 to 136 electrons and having masses up to 7×10^{-10} grams. He found that, within 1/10 of 1 per cent., the charge on these drops was always an integral multiple of $e = 4.774 \times 10^{-10}$ esu. Now, since there are about 6×10^{23} protons in a gram of matter, these drops contained 4×10^{14} protons and very nearly the same number of electrons. Hence, since each drop, whatever num-

[1] Aston, Phil. Mag., 38 (1919), p. 317.

[2] Millikan, Phil. Mag., 34 (1917), p. 1. The Electron, p. 109.

ber of protons it contained, could be made exactly neutral within 1/1000 of the electronic charge, it follows that the ratio of the average charge on a proton to the average charge on an electron is equal to -1 to one part in $1000 \times 4 \times 10^{14}$, *i.e.*, to $-1 \pm 2.5 \times 10^{-18}$. Moreover it would seem a highly improbable coincidence if the charges of protons or electrons should vary among different classes, being different say, in carbon from in hydrogen, or if the charge of the peripheral electrons should differ from those in the K ring or the nucleus, yet in such a way that the average charge on the proton came out exactly equal to the average charge on the electron. So that this experiment may also be considered a precise confirmation of the relativity law of the invariance of electric charge.[1]

Probably no physical constant is more accurately known than this ratio. The precision of the measurement arises, somewhat as in interference measurements, from the fact that one is able to compare the charge on a large and necessarily integral number (4×10^{14}) of protons with a similar number of electrons to a precision of 1/1000 of a single charge. Millikan[2] considered the evidence that his experiments afforded on the equality of positive and negative elementary charges. But, writing at a time when Aston's work had not yet identified the universal atom of positive electricity, or proton, and when it was natural to think in terms of a single substance theory of electricity, he merely stated that the positive and negative charges acquired by his drops were all integral multiples of e, and hence "the mean value of the positive electron agrees with the mean value of the negative electron to within less than 1 part in 2000." He was not then in a position to count up the total number of positive and negative elementary charges and state that they were equal to within one part in 4×10^{17}.

It is at present most convenient to think of matter, or electricity, as made up of two fundamental entities, corresponding to protons and electrons. But the exactness of this simple relation between them, analogous to that between inertia and gravitation, is convincing evidence of an underlying unity.

All this does not in any way affect the theory which attributes gravitation to a slight excess in the attraction between charges of opposite sign over the repulsion between charges of the same sign. Which

[1] Thus, if the velocities of extra-nuclear electrons can be calculated from X-ray data by the Bohr theory, it follows from the fact that the above phenomenon is observed both with oil and mercury drops that, if the charge is proportional to $(1-v^2/c^2)^n$, then $|n| < 10^{-14}$; or if the charge is expressible as a power series in v/c, then the first term which can have a coefficient as great as 0.001 is the 48th. Still more exact statements can be inferred from any of the usual assumptions about the velocities of nuclear electrons.

[2] Millikan, The Electron, pp. 80–83.

theory, in view of the now recognized identity between electricity and matter, is really not an explanation of gravity at all, but merely an unexceptionable statement that the net force between particles is the sum of the electric and gravitational forces. Even the precision of Millikan's measurements is not enough to detect this excess gravitational force, as one can see without computations; for the effect to be expected is essentially the *gravitational* attraction between the oil drop and the electrons which are driven around from one condenser plate to the other when the electric field is put on. Obviously this is quite beyond the range of Millikan's measurements, which would have to be accurate to one part in 10^{40} to show it.

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RADIATION FROM A GROUP OF ELECTRONS.

BY LEIGH PAGE.

SYNOPSIS.

Electric Intensity and Radiation Flux at a Great Distance from a Group of Electrons.—(1) The *general equation* for the intensity is obtained from the field equations of classical electrodynamics by expanding the retarded expression for the intensity due to a point charge:

$$\boldsymbol{E} = \frac{1}{4\pi Rc^2}\left[\left\{\frac{d}{dt}(\Sigma e\boldsymbol{v}) + \frac{1}{c}\frac{d^2}{dt^2}(\Sigma e\boldsymbol{r}\cdot\boldsymbol{M}\boldsymbol{v}) \cdots + \frac{1}{(n-1)!c^{n-1}}\frac{d^n}{dt^n}(\Sigma e\boldsymbol{r}\cdot\boldsymbol{M}^{n-1}\boldsymbol{v}) + \cdots\right\} \times \boldsymbol{M}\right] \times \boldsymbol{M},$$

where R is the distance from a fixed point in the group, $\boldsymbol{r}$ is the position vector of each electron, $\boldsymbol{v}$ its velocity, and $\boldsymbol{M}$ a unit vector in the direction of R. The Poynting flux is merely $c\boldsymbol{E}^2$. The terms in the series correspond to successive orders of radiation. The general condition for the absence of the nth order of radiation is evidently that $\Sigma e\boldsymbol{r}\cdot\boldsymbol{M}^{n-1}\boldsymbol{v}$ = constant. (2) *For a ring of evenly spaced electrons,* the radiation of all orders less than k, the number of electrons, vanishes; but the radiation of order k does not. In fact $R_k = C_k\beta^{2(k-1)}R_1$, where β is the speed relative to that of light and C_k is a constant which increases rapidly with k, so that for most atoms the higher order radiation from a ring with a number of electrons should be even greater than the radiation from the single electron of the hydrogen atom. Only in the case of a continuous distribution of electricity along the ring do all orders vanish.

Radiationless orbits of Bohr's atomic theory are seen to be in definite conflict with classical electrodynamics. Even the rotation of an asymmetrical positive nucleus would not neutralize the radiation from the hydrogen electron. Therefore, the advisability of seeking some other interpretation of the numerical relationships established by the theory is suggested.

CLASSICAL electrodynamics requires an irreversible radiation of energy from a solitary electron whenever it is accelerated. If, however, a number of electrons at distances from one another small compared to the wave-length emitted are accelerated, the radiations from the individual electrons may interfere in such a way as to annul the first order radiation from the group. This is the case when two or more evenly spaced electrons revolve in a single circular orbit. When the velocities of the electrons are high, however, the radiation of lowest order which does not vanish may be quite comparable with the first order radiation which would be emitted if all the electrons save one were removed. Therefore it has seemed worth while to investigate the higher order radiation from a group of electrons as required by the classical electrodynamics.

Choose as origin some point O in the group of electrons. Describe about O a sphere of radius R very large compared to the greatest distance between electrons in the group. Let $\boldsymbol{c}$ be a vector equal in magnitude to the velocity of light, and having a direction from O toward a point P on the surface of the sphere. The Poynting flux at P is

$$\boldsymbol{s} = c(\boldsymbol{E} \times \boldsymbol{H}).$$

But

$$\boldsymbol{H} = \frac{1}{c}(\boldsymbol{c} \times \boldsymbol{E}),$$

and, as both $\boldsymbol{E}$ and $\boldsymbol{H}$ are at right angles to $\boldsymbol{c}$,

$$\boldsymbol{s} = c\boldsymbol{E}^2. \tag{1}$$

The retarded electric intensity at a distance R from an electron[1] is

$$\boldsymbol{E} = \frac{e}{4\pi R^2 k^2 c\left(1 - \frac{\boldsymbol{c}\cdot\boldsymbol{v}}{c^2}\right)^3}\left[(\boldsymbol{c} - \boldsymbol{v}) + \frac{Rk^2}{c^3}\{\boldsymbol{f} \times (\boldsymbol{c} - \boldsymbol{v})\} \times \boldsymbol{c}\right]$$

in Heaviside-Lorentz rational units, where $\boldsymbol{v}$ is the velocity of the electron, $\boldsymbol{f}$ its acceleration, and $k^2 \equiv 1 - \beta^2$, where $\beta \equiv v/c$. By making R great enough, the first term can be made as small as desired compared to the second, and hence at P

$$\boldsymbol{E} = \frac{e}{4\pi R c^4\left(1 - \frac{\boldsymbol{c}\cdot\boldsymbol{v}}{c^2}\right)^3}\{\boldsymbol{f} \times (\boldsymbol{c} - \boldsymbol{v})\} \times \boldsymbol{c}. \tag{2}$$

Let $\boldsymbol{r}$ be the position vector of an electron in the group relative to the origin O at the time 0. Denote by $\boldsymbol{M}$ a unit vector in the direction of $\boldsymbol{c}$. Let $\boldsymbol{r}_e$ be the position vector of the electron at the time $\boldsymbol{r}_e \cdot \boldsymbol{M}/c$. Then $\boldsymbol{r}_e$ specifies the *effective position* of the electron. The radiations emitted by all the electrons when in their effective positions will reach P at the same time R/c. The electric intensity involved in this radiation will now be calculated.

Replacing $\boldsymbol{c}$ in (2) by $c\boldsymbol{M}$, and expanding, the electric intensity at P due to a single electron can be put in the form

$$\boldsymbol{E} = \frac{e}{4\pi Rc^2}\left[\left\{\frac{\boldsymbol{f}_e}{(1 - \beta_e\cdot\boldsymbol{M})^2} + \frac{\boldsymbol{f}_e\cdot\boldsymbol{M}}{c}\frac{\boldsymbol{v}_e}{(1 - \beta_e\cdot\boldsymbol{M})^3}\right\} \times \boldsymbol{M}\right] \times \boldsymbol{M}$$

$$= \frac{e}{4\pi Rc^2}\left[\left\{\frac{1}{1 - \beta_e\cdot\boldsymbol{M}}\frac{d}{dt_e}\left(\frac{\boldsymbol{v}_e}{1 - \beta_e\cdot\boldsymbol{M}}\right)\right\} \times \boldsymbol{M}\right] \times \boldsymbol{M}.$$

[1] See the author's "Introduction to Electrodynamics," p. 29, or Abraham, "Theorie der Electrizität," Vol. II., p. 99.

Now,

$$t_e = \frac{\mathbf{r}_e \cdot \mathbf{M}}{c},$$

and after a time dt has elapsed at O (Fig. 1),

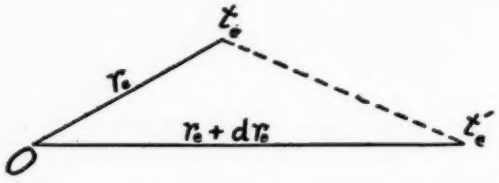

Fig. 1.

$$t_e' = dt + \frac{\mathbf{r}_e \cdot \mathbf{M}}{c} + \beta_e \cdot \mathbf{M} dt_e.$$

Hence

$$\begin{aligned} dt_e &\equiv t_e' - t_e \\ &= dt + \beta_e \cdot \mathbf{M} dt_e, \end{aligned}$$

or

$$\frac{1}{1 - \beta_e \cdot \mathbf{M}} \frac{d}{dt_e} = \frac{d}{dt}. \tag{3}$$

Therefore

$$\begin{aligned} \mathbf{E} &= \frac{e}{4\pi R c^2}\left[\left\{\frac{d}{dt}\left(\frac{\mathbf{v}_e}{1 - \beta_e \cdot \mathbf{M}}\right)\right\} \times \mathbf{M}\right] \times \mathbf{M} \\ &= \frac{e}{4\pi R c^2}\left[\left\{\frac{d}{dt}\left(\frac{1}{1 - \beta_e \cdot \mathbf{M}} \frac{d\mathbf{r}_e}{dt_e}\right)\right\} \times \mathbf{M}\right] \times \mathbf{M} \\ &= \frac{e}{4\pi R c^2}\left[\frac{d^2 \mathbf{r}_e}{dt^2} \times \mathbf{M}\right] \times \mathbf{M}. \end{aligned} \tag{4}$$

Put

$$\tau = \frac{\mathbf{r} \cdot \mathbf{M}}{c}, \qquad \tau_e = \frac{\mathbf{r}_e \cdot \mathbf{M}}{c}.$$

Then

$$\mathbf{r}_e = \mathbf{r} + \mathbf{v}\tau_e + \frac{1}{2}\dot{\mathbf{v}}\tau_e^2 \cdots + \frac{1}{(n-1)!}\mathbf{v}^{(n-2)}\tau_e^{n-1} \cdots \tag{5}$$

and

$$\begin{aligned} \tau_e &= \tau + \dot{\tau}\tau_e + \frac{1}{2}\ddot{\tau}\tau_e^2 \cdots + \frac{1}{(n-1)!}\tau^{(n-1)}\tau_e^{n-1} \cdots \\ \tau_e^2 &= \tau^2 + \dot{\tau^2}\tau_e + \frac{1}{2}\ddot{\tau^2}\tau_e^2 \cdots + \frac{1}{(n-1)!}\tau^{2(n-1)}\tau_e^{n-1} \cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \tau_e^m &= \tau^m + \dot{\tau^m}\tau_e + \frac{1}{2}\ddot{\tau^m}\tau_e^2 \cdots + \frac{1}{(n-1)!}\tau^{m(n-1)}\tau_e^{n-1} \cdots \end{aligned} \tag{6}$$

The solution of (6) is

$$\tau_e^m = \tau^m + \frac{m}{m+1}\dot{\tau}^{m+1} + \frac{m}{2\cdot m+2}\ddot{\tau}^{m+2} \cdots + \frac{m}{(n-1)!\,m+n-1}\tau^{m+n-1(n-1)} \cdots \quad (7)$$

The correctness of this solution may be proved by mathematical induction. For assume that the solution has this form for the first n terms of τ_e, for the first $n-1$ terms of τ_e^2, and in general for the first $n-m+1$ terms of τ_e^m for all m's less than $n+1$. Then by substituting the assumed terms in (6), the $n-m+2$ term of τ_e^m is found to be

$$\frac{1}{(n-m+1)!}\left[\tau^{m(n-m+1)}\tau^{n-m+1} + (n-m)\tau^{m(n-m)}\tau^{n-m+1(1)} + \text{etc.}\right],$$

which, by Leibnitz's theorem for the kth derivative of the product of two factors, is equal to

$$\frac{1}{(n-m+1)!}\frac{d^{n-m}}{dt^{n-m}}\left[\frac{d}{dt}(\tau^m)\tau^{n-m+1}\right]$$

$$= \frac{m}{(n-m+1)!}\frac{d^{n-m}}{dt^{n-m}}\left[\tau^n\dot{\tau}\right]$$

$$= \frac{m}{(n-m+1)!\,n+1}\tau^{n+1(n-m+1)},$$

which is the form predicted by (7) for the $n-m+2$ term.

Now the first term of τ_e is obviously τ. Hence all the terms of all the series are as given by (7).

Substituting in the expression (5) for r_e the values of τ_e^m given by (7), the nth term of r_e is found to be

$$\frac{1}{(n-1)!}\left[v^{(n-2)}\tau^{n-1} + (n-2)v^{(n-3)}\tau^{n-1(1)} + \text{etc.}\right] = \frac{1}{(n-1)!}\frac{d^{n-2}}{dt^{n-2}}(v\tau^{n-1}).$$

Hence

$$\tau_e = \tau + v\tau + \frac{1}{2}\frac{d}{dt}(v\tau^2)\cdots + \frac{1}{(n-1)!}\frac{d^{n-2}}{dt^{n-2}}(v\tau^{n-1})\cdots \quad (8)$$

and substitution in (4) gives the electric intensity due to a single electron at a great distance R from the group in the direction M. Summing up

over all the electrons, and replacing τ by $\boldsymbol{r}\cdot\boldsymbol{M}/c$, the total electric intensity at P is seen to be

$$\boldsymbol{E} = \frac{1}{4\pi Rc^2}\left[\left\{\frac{d}{dt}(\Sigma e\boldsymbol{v}) + \frac{1}{c}\frac{d^2}{dt^2}(\Sigma e\boldsymbol{r}\cdot\boldsymbol{M}\boldsymbol{v}) + \frac{1}{2c^2}\frac{d^3}{dt^3}(\Sigma e\boldsymbol{r}\cdot\overline{\boldsymbol{M}^2}\boldsymbol{v}) \right.\right.$$
$$\left.\left. \cdots + \frac{1}{(n-1)!\,c^{n-1}}\frac{d^n}{dt^n}(\Sigma e\boldsymbol{r}\cdot\overline{\boldsymbol{M}^{n-1}}\boldsymbol{v}) \cdots \right\} \times \boldsymbol{M}\right] \times \boldsymbol{M}. \qquad (9)$$

Now (1) shows that the condition for absence of radiation is that $\boldsymbol{E}$ shall vanish for all directions of $\boldsymbol{M}$. If the electrons are to remain in a group, and not become separated further and further as time goes on, this condition requires that each of the vector sums enclosed in parentheses in (9) shall remain constant in time. Therefore in scalar notation the conditions for absence of radiation of each order are:

Order of Radiation.	Conditions.	No. of Conditions for Three-dimensional Motion.	No. of Conditions for Two-dimensional Motion.
1st.....	$\Sigma e\dot{x}_j$ = const.	3	2
2d.....	$\Sigma e x_a \dot{x}_j$ = const.	9	4
3d.....	$\Sigma e x_a x_b \dot{x}_j$ = const.	18	6
nth.....	$\Sigma e x_a x_b \cdots x_{n-1}\dot{x}_j$ = const.	$\frac{3}{2}n(n+1)$	$2n$

where x_1, x_2, x_3 are rectangular coördinates, and the suffixes a, b, $c\cdots$ assume values corresponding to all independent combinations of 1, 2, and 3, j having successfully the values 1, 2, and 3. As Larmor[1] has pointed out, a necessary but not sufficient condition for absence of second order radiation is that the magnetic moment of the group remain constant.

An interesting application of the conditions for absence of radiation can be made to the case of k evenly spaced electrons revolving in a fixed circular orbit with constant speed. The conditions for absence of radiation of the nth order become

$$e\Omega a^n \sum_p \sin^\alpha(\theta + P)\cos^\beta(\theta + P) = \text{const.}, \qquad (10)$$

where Ω is the common angular velocity of the electrons, a the radius of the orbit, α and β any positive integers whose sum is n, θ the angular displacement of the first electron, and

$$P \equiv \frac{2\pi}{k}(p-1),$$

the summation over the electrons extending from $p = 1$ to $p = k$.

[1] Phil. Mag., *42*, 595, 1921.

Writing the sine and cosine in exponential form, expanding by the binomial theorem and multiplying, each term in (10) is easily seen to have the form

$$C\sum_p e^{i[n-2(s+t)](\theta+P)} = f(\theta)\frac{1 - e^{2\pi i[n-2(s+t)]}}{1 - e^{\frac{2\pi}{k}i[n-2(s+t)]}}$$

where s and t are positive integers such that

$$s + t \leqq n,$$

and C is a constant not containing θ.

When $n - 2(s + t)$ is zero, the term reduces to the constant pC, which is not a function of θ and hence does not vary with the time. Otherwise each term vanishes except when $n - 2(s + t)$ is equal to an integral multiple of k. But this cannot occur if n is less than k. Hence *the radiation of all orders less than the number of electrons in the ring vanishes.* Thus, if there are two electrons in the ring, the first order radiation will vanish; if there are eight electrons, radiation of the first seven orders will vanish.

Suppose that the number k of electrons in the ring is even. Then all odd orders of radiation vanish, for $n - 2(s + t)$ cannot equal k for n odd. Similarly, if the number of electrons in the ring is odd, all even orders of radiation less than $2k$ vanish.

If the number of electrons in the ring is infinite, the radiation of all orders vanishes. Hence a continuous ring of electricity of uniform density rotating about its axis of symmetry will not radiate—a well-known result.[1]

The radiation emitted by any specified group of electrons may easily be calculated from (9) and (1). Thus the total radiation of lowest order emitted by a pair of electrons revolving in a circle so as to be always at opposite ends of a diameter, is the second order radiation. In Heaviside-Lorentz rational units, it amounts to

$$R_2 = 9.6\beta^2 R_1, \tag{11}$$

where

$$R_1 \equiv \frac{e^2 f^2}{6\pi c^3}$$

is the first order radiation that would be emitted by one of the two electrons in the absence of the other. Therefore the second order radiation from a pair of electrons is greater than the first order radiation from a

[1] Schott, Phil. Mag., *36*, 255, 1918.

single electron traversing the same orbit at the same rate if

$$\beta > .323.$$

On Bohr's theory of the atom this occurs in the K rings of elements of atomic number greater than 44 (ruthenium). Furthermore, a simple calculation shows that the second order radiation from the pair of electrons in the K rings of elements of atomic number greater than 2 (helium) should be greater than the first order radiation from the single electron in the hydrogen atom when it is in the first Bohr ring.

In general, if k evenly spaced electrons are revolving in a circular orbit, the radiation of the kth order (which is the lowest order which does not vanish) is

$$R = C_k \beta^{2(k-1)} R_1,$$

where the numerical constant C_k increases rather rapidly with k, being 9.6 for two electrons and 173 + for four electrons.

The chief object of this paper is to emphasize the inconsistency between Bohr's theory of the atom and classical electrodynamics. In the case of the hydrogen or ionized helium atom, where the theory has been most successful, Bohr requires the single electron to revolve about the nucleus in a circular or elliptical orbit without radiating. Electrodynamics, on the other hand, allows no possibility of orbital motion of a solitary electron without radiation. Even in the case of a number of electrons following one another around the same orbit, the higher order radiation emitted should be greater, in the case of most atoms, than the first order radiation from hydrogen. Now the radiation of energy specified by the Poynting vector is one of the most fundamental properties of classical electrodynamics. Hence it is difficult to see how electrodynamics and Bohr's theory of the atom can ever find a common ground. This raises the question of the advisability of seeking for other than a dynamical interpretation of the numerical relationships which Bohr's theory has been so successful in establishing.

It might be thought that the radiation from the single electron in the hydrogen or ionized helium atom might be annulled by radiation from an symmetrical nucleus rotating at the same rate as the electron. Suppose, for instance, that the hydrogen nucleus consists of two small coincident circular rings of electricity, one positive and the other negative, rotating in opposite senses about their common center, and having charges of such densities that the net charge on the nucleus is one unit of positive electricity. Investigation shows that the asymmetric distribution of charge and the angular velocity of rotation can be chosen so

as to satisfy the conditions of each order for absence of radiation from the atom. But the equal quantities of positive and negative electricity which must be distributed asymmetrically on the two rings to annul the radiation of each order become greater and greater as the order increases, as indeed is evident from the fact that the exponent of $\boldsymbol{r}\cdot\boldsymbol{M}$ in the terms of (9) increases with the ordinal number of the term. Hence the presence of infinite amounts of both positive and negative electricity within the nucleus would be required in order to make possible the entire annulment of radiation from the atom—a condition which makes the formal solution of the problem of no physical value.

Sloane Physics Laboratory,
Yale University,
Jan. 26, 1922.

ANOTHER ETHER-DRIFT EXPERIMENT.

BY ROY J. KENNEDY.

SYNOPSIS.

Introduction.—The ether drift experiments heretofore performed seem to have proven the reality of the Lorentz-Fitzgerald contraction, but the explanation of the negative results requires nothing further. The experiment forming the subject of this paper is designed to detect ether drift or to confirm the time transformation between relatively moving systems, which is an essential part of the theory of relativity.

Theory.—Assuming Huyghen's principle and the Lorentz-Fitzgerald contraction it can be shown that the paths of light rays relative to a system are unaffected by its motion or orientation in the medium transmitting the light. The relative phases of two homogeneous beams superposed under large retardation are unaffected by the orientation but not by the velocity of the system. The effect sought in this experiment depends on the velocity change due to the alternate adding and subtracting of the earth's orbital motion and that of the solar system as a whole.

Apparatus and Method.—A delicate optical system is used in which a half-shade analyzer measures the variation with change of phase of component beams, of the position of the plane of polarization of a beam resulting from the superposition of two oppositely circularly polarized ones. The change manifests itself as an unbalancing of a uniform photometric field. The smallest effect to be expected according to the classical ether theory is shown to be several times the least detectable with the experimental arrangements, so that conclusive evidence for or against the relativity principle should be obtained.

Introduction.—The negative results of all experiments devised to detect relative motion between the earth and the ether, which gave rise to the theory of relativity, can be explained without modification of the classical electromagnetic theory if the forces on which the size and shape of rigid bodies depend are assumed to be electrical. For then the Lorentz-Fitzgerald contraction of the experimental apparatus follows necessarily, and nothing further than the contraction is required to account for the absence of any physical effect of motion with the arrangements heretofore used. For this reason the hypothesis of the relativity of time, which is essential to the theory of relativity, appears to be a gratuitous assumption. This paper is an outline of an experiment which is intended to detect ether-motion or, failing that, to establish the validity of the Lorentz-Einstein time transformation. It is being performed by the writer at the Palmer Physical Laboratory.

Apparatus.—The arrangement of apparatus is as shown in Fig. 1. A beam of homogeneous plane-parallel light polarized in a plane inclined

at an angle of 45° to that of the paper falls on a Wollaston prism W which divides it into two beams of equal intensity and plane polarized in planes at right angles to one another. The one whose plane of polarization is in the plane of the paper passes to the mirror M_1, from which it is reflected to mirror M_2, and thence to a partially silvered mirror M_3. Here

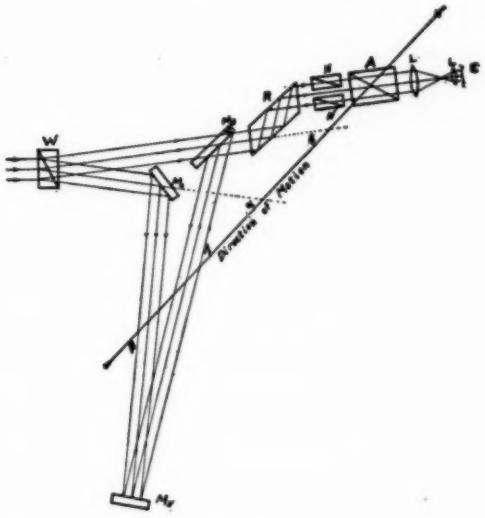

Fig. 1.

it is reflected and superposed on the other ray, which is polarized in a plane at right angles to the paper and passes through M_3. The elliptically polarized beam resulting from this superposition passes through a Fresnel rhomb R, so placed as to render it plane polarized, the inclination of the plane of polarization depending on the phase relation between the two beams combining at M_3. From R the beam passes between and through the small nicols NN, whose principal sections are parallel, through the analyzing nicol A and to the eye at E by way of the lens system LL. The nicols NN are fixed in position, while the nicol A is rotatable about an axis in the direction of the rays through it, its position being read off a graduated circle. The optical parts are mounted so that temperature changes are compensated for and do not affect the paths of the rays.

Such an arrangement provides a very sensitive measure of any change in relative phase of the two superposed beams. It can be shown that if the plane of polarization of the light emerging from the rhomb R makes an angle θ with the planes of the principal sections of the nicols NN, and ψ is the angle between the principal sections of NN and of A when the latter prism is set so that it lets through equal amounts of the light

passed through and between NN, then to a small change $\delta\theta$ in θ there corresponds a change in ψ

$$\delta\psi = \frac{2\,\delta\theta}{1 + 3\cos^2\theta}.$$

Since for the greatest sensibility θ must be small (not greater than two or three degrees), $\delta\psi$ will be about half of $\delta\theta$. The light emerging from R may be regarded as the resultant of two oppositely circularly polarized beams; a little consideration will show that its plane will rotate through 180° as the relative phase of the component beams varies through 360° or one wave. With a sufficiently intense source of light the analyzer A can be set accurately to one-hundredth of a degree; that is, a change $\delta\psi = 0.01$ can be detected. Therefore a change in the phase ϕ of

$$\delta\phi = \frac{2 \times .01^\circ}{180^\circ} = \frac{1}{9000}\text{ wave}$$

can be detected. An ordinary Michelson or Fabry and Perot interferometer could be used but for their low sensibility.

Theory.—Assuming the existence of an ether with respect to which the surface of the earth and the apparatus are in motion with a velocity v, and the Lorentz-Fitzgerald contraction which reduces dimensions in the direction of motion in the ratio $\sqrt{1 - \frac{v^2}{c^2}} : 1$ (where c is the velocity of light in ether), the theory of the experiment is as follows:

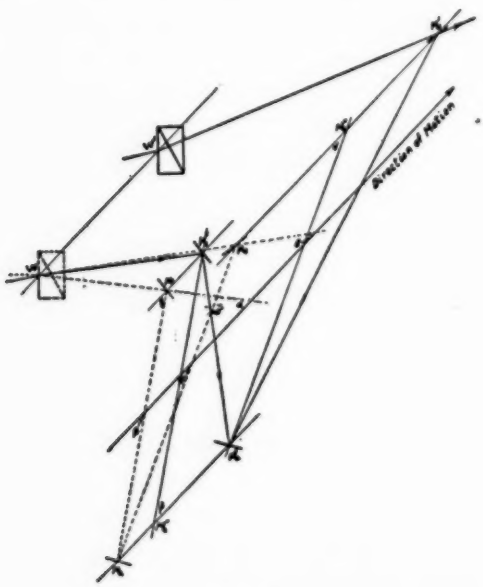

Fig. 2.

In Fig. 2 the dotted broken lines, $WM_1M_1M_3$ and WM_3 represent the paths of the two beams relative to the apparatus. The full lines $WM_1'M_2'M_3'$ and $W'M_3'$ are the paths as referred to the ether. The dotted line sketch of the Wollaston prism and the mirrors shows them in the position occupied at the instant the lower beam leaves the prism. The full line sketches show the positions of the mirrors as they are successively reached by the wave front, and of the prism at the instant of starting of the upper train which will combine at M_3' with the lower. M_2'' is position of mirror M_2 at the instant when the wave front starts from M_1' and M_3'' is that of the mirror M_3 when the wave front leaves M_2'.

Let the lengths WM_1 equal l_1, M_1M_2 equal l_2, M_2M_3 equal l_3, and WM_3 equal l_4, these being the values which would be measured by an observer stationary in the ether — the "contracted" values. Let the direction of motion makes angles α with WM_1, β with M_1M_2, γ with M_2M_3 and δ with WM_3. Consider first the ray from the prism to the mirror M_1. Due to the motion of the system, its actual path in the ether will be along WM_1' instead of WM_1, and if the time required for the light to travel this distance is t_1, we have from the geometry of the figure,

$$\begin{aligned} c^2t_1^2 &= \overline{WM_1}^2 + \overline{M_1M_1'}^2 - 2\,\overline{WM_1}\,\overline{M_1M_1'}\cos < WM_1M_1' \\ &= l_1^2 + v^2t_1^2 - 2\,l_1vt_1\cos(\pi - \alpha) \\ &= l_1^2 + v^2t_1^2 + 2l_1vt_1\cos\alpha. \end{aligned}$$

After reflection at M_1' the light travels to M_2' in time t_2 a distance whose square is

$$\begin{aligned} c^2t_2^2 &= \overline{M_1'M_2'}^2 = \overline{M_1'M_2''}^2 + \overline{M_2''M_2'}^2 - 2\,\overline{M_1'M_2''}\,\overline{M_2''M_2'}\cos\beta \\ &= l_2^2 + v^2t_2^2 - 2\,l_2vt_2\cos\beta. \end{aligned}$$

After reflection at M_2' the ray proceeds to M_3' in time t_3 a distance whose square is

$$\begin{aligned} c^2t_3^2 &= \overline{M_2'M_3'}^2 = \overline{M_2'M_3''}^2 + \overline{M_3''M_3'}^2 - 2\,\overline{M_2'M_3''}\,\overline{M_3''M_3'}\cos(\pi-\gamma) \\ &= l_3^2 + vt_3^2 + 2\,l_3vt_3\cos\gamma. \end{aligned}$$

At M_3' the ray combines with one directly from W' which has gone in time t_4 a distance given by

$$c^2t_4^2 = \overline{W'M_3'}^2 = l_4^2 + v^2t_4^2 + 2\,l_4vt_4\cos\delta.$$

Solving these four equations and rejecting one physically meaningless solution of each we obtain

$$t_1 = \frac{l_1}{c^2 - v^2}\left(v\cos\alpha + c\sqrt{1 - \frac{v^2}{c^2}\sin^2\alpha}\right),$$

$$t_2 = \frac{l_2}{c^2 - v^2}\left(- v\cos\beta + c\sqrt{1 - \frac{v^2}{c^2}\sin^2\beta}\right),$$

$$t_3 = \frac{l_3}{c^2 - v^2}\left(v\cos\gamma + c\sqrt{1 - \frac{v^2}{c^2}\sin^2\gamma}\right),$$

$$t_4 = \frac{l_4}{c^2 - v^2}\left(v\cos\delta + c\sqrt{1 - \frac{v^2}{c^2}\sin^2\delta}\right).$$

If we represent by primed letters the distances between pairs of points in the system when it is at rest (which are the distances actually obtained by measurement by an observer moving with the system)then any length $l = l'\sqrt{1 - (v^2/c^2)\cos^2\omega}$ where ω is the angle between direction of the motion and the line joining the two points. For the component l_y of the length perpendicular to the motion is unaffected while the parallel component l_x is contracted to $\sqrt{1 - (v^2/c^2)}$ times its value when stationary, and

$$l = \sqrt{l_y^2 + l_x^2} = \sqrt{(l'\sin\omega)^2 + \left(l'\sqrt{1 - \frac{v^2}{c^2}\cos\omega}\right)^2}$$

$$= l'\sqrt{\sin^2\omega + \cos^2\omega - \frac{v^2}{c^2}\cos^2\omega}$$

$$= l'\sqrt{1 - \frac{v^2}{c^2}\cos^2\omega}.$$

Therefore

$$l_1 = l_1'\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha}, \qquad l_3 = l_3'\sqrt{1 - \frac{v^2}{c^2}\cos^2\gamma},$$

$$l_2 = l_2'\sqrt{1 - \frac{v^2}{c^2}\cos^2\beta}, \qquad l_4 = l_4'\sqrt{1 - \frac{v^2}{c^2}\cos^2\delta}.$$

Substituting these values in formulas (1)

$$t_1 = \frac{l_1'}{c^2 - v^2}\left(v\cos\alpha\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha} + c\sqrt{1 - \frac{v^2}{c^2}\sin^2\alpha - \frac{v^2}{c^2}\cos^2\alpha + \frac{v^4}{c^4}\sin^2\alpha\cos^2\alpha}\right)$$

$$= \frac{l_1'}{c^2 - v^2}\left(v\cos\alpha\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha} + c\sqrt{1 - \frac{v^2}{c^2}}\right),$$

neglecting the fourth order term $(u^4/c^4)\sin^2\alpha\cos^2\alpha$. Similar relations hold for t_2, t_3 and t_4 so that the retardation of the reflected ray relative to the direct ray may be written

$T = t_1 + t_2 + t_3 - t_4$

$$= \frac{1}{c^2 - v^2}\Bigg[v\Bigg(l_1'\sqrt{1 - \frac{v^2}{c^2}\cos^2\alpha}\cos\alpha - l_2'\sqrt{1 - \frac{v^2}{c^2}\cos^2\beta}\cos\beta + l_3'\sqrt{1 - \frac{v^2}{c^2}\cos^2\gamma}\cos\gamma - l_4'\sqrt{1 - \frac{v^2}{c^2}\cos^2\delta}\cos\delta\Bigg) + c\sqrt{1 - \frac{v^2}{c^2}}\,(l_1' + l_2' + l_3' - l_4')\Bigg].$$

The coefficient of v in this expression is zero, since it is the algebraic sum of the projections of the sides of a closed figure on a line (the direction of motion). Hence

$$T = \frac{c\sqrt{1 - \frac{v^2}{c^2}}\,(l_1' + l_2' + l_3' - l_4')}{c^2 - v^2},$$

$$= \frac{1}{c\sqrt{1 - \frac{v^2}{c^2}}}(l_1' + l_2' + l_3' - l_4').$$

As would be expected, this retardation is not a function of the orientation of the system; this is indicated by the absence of α, β, γ and δ from the expression for T. Its value does, however, depend on the velocity of the system. For another value v' of the velocity, the new retardation

$$T' = \frac{1}{c\sqrt{1 - \frac{v'^2}{c^2}}}(l_1' + l_2' + l_3' - l_4')$$

and

$$T - T' = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}}\right)\frac{l_1' + l_2' + l_3' - l_4'}{c}$$

$$= \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} \cdots\cdots - 1 - \frac{1}{2}\frac{v'^2}{c^2} - \frac{3}{8}\frac{v'^4}{c^4} \cdots\cdots\right)\frac{l_1' + l_2' + l_3' - l_4'}{c}$$

$$= \frac{v^2 - v'^2}{2c^2}\left(l_1' + l_2' + l_3' - l_4'\right)$$

to a very close approximation. This quantity is the difference between the time-retardations for the two velocities. The change $\delta\phi$ in the number of waves corresponding to this time is T—T' multiplied by the frequency of the light employed. But the frequency equals c/λ, so that

$$\delta\phi = \frac{c}{\lambda}\left(T - T'\right) = \frac{v^2 - v'^2}{2c^2}\left(\frac{l_1' + l_2' + l_3' - l_4'}{\lambda}\right).$$

Up to the last step, nothing in the argument is contrary to the relativity hypothesis if the times T and T' be taken as determined by an observer stationary in the ether. The departure is in assuming λ to be the same for both velocities, v and v'. For according to the relativity time transformation, the indication of a clock in one system during the interval between two events, as observed from the other system, is equal to $\sqrt{1-(v^2/c^2)}$ times the indication of a clock in the latter between the same events. To an observer stationary in the ether, then, any time indicator in the apparatus of this experiment should run slow in the above ratio. A luminous emitter is a form of clock of which the indication is expressed as a certain number of waves. If its rate decreases to $\sqrt{1-(v^2/c^2)}$ times its rest value, the wave length corresponding to a given spectral line will increase in the inverse ratio; *i.e.*, if λ_0 be the wave length corresponding to the line when the system is at rest in the ether, the wave length for a velocity v will be $\dfrac{\lambda_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

If this value is substituted in the expression for the retardation in waves ($= T$ times the frequency) we have

$$\frac{Tc}{\lambda} = \frac{c}{\dfrac{\lambda_0}{\sqrt{1-\dfrac{v^2}{c^2}}}\,c\sqrt{1-\dfrac{v^2}{c^2}}}\,(l_1' + l_2' + l_3' - l_4') = \frac{l_1' + l_2' + l_3' - l_4'}{\lambda_0}.$$

This is quite independent of v and would predict a negative result for the experiment.

The effect to be expected if the frequency is not affected by the motion is well within the range of the apparatus. It has been found possible to secure interference with a retardation (equal to $(l_1' + l_2' + l_3' - l_4')/\lambda$) of over a million waves, so it is probable that half that retardation will be found feasible. It was shown above that a value of $\delta\phi$ equal to $\frac{1}{2000}$ wave could be detected under good conditions. As a conservative estimate let us assume that about $\frac{1}{2000}$ wave is the least value under the less favorable condition of high retardation that will obtain in the experiment. The least difference of the squares of the velocities (in miles per second) detectable will then be

$$V^2 - V'^2 = \frac{2c^2\,\delta\phi}{\left(\dfrac{l_1' + l_2' + l_3' - l_4'}{\lambda}\right)} = \frac{2 \times (186{,}000)^2 \times .0005}{500{,}000}$$

$$= 69, \text{ approximately.}$$

This is about four times the value of the diurnal difference of squares if the earth is supposed to have a velocity with respect to the ether equal to its orbital velocity merely (the least that can be ascribed to it). If the velocity is twice this at some time of the year the effect would be just observable. The yearly variation due to the motion of the solar system as a whole should be many times as great. If this latter speed is no greater than that towards the constellation Hercules — which is supposed to be the average velocity with respect to the whole sidereal system — which has a maximum component in the earth's orbital plane of slightly more than six miles per second, the maximum and minimum values of the velocity are $18 + 6$ and $18 - 6$, so that $V^2 - V'^2 = 24^2 - 12^2 = 432$. It is quite probable that the actual velocity is much greater.

If the experiment does show relative motion between earth and ether, it will be possible to determine the absolute value of it. For the difference between the first powers of the two velocities will be known from the earth's orbital and rotational velocities, and the experiment will give the difference between their squares, thus supplying the two equations necessary to determine two unknowns. If no positive effect is obtained, the relativity time transformation between a system stationary in the ether and one moving with respect to it would seem to be confirmed.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
February, 1922.

NOTE ON THE LORENTZIAN ELECTRON.

BY I. MAIZLISH.

SYNOPSIS.

The Effect of a Uniform Dilation of the Lorentzian Electron.

Assuming that, in addition to the contraction *in the line of motion*, the Lorentzian electron experiences a *uniform* expansion proportional to a function, $\lambda(\beta)$, of the ratio of its velocity to that of light, the author derives general expressions for the longitudinal and transverse momenta and masses, and shows that a dilation equal to

$$\frac{G\beta}{G\beta+\delta\sqrt{1-\beta^2}}$$

would leave the longitudinal mass unaltered whatever G and δ may be. But if *both* masses are to be identical with those given by the formulæ of Lorentz, then there can be no uniform dilation of the electron.

1. It is usually assumed that the Lorentzian electron suffers only a contraction *in the line of motion*. Suppose that, in addition to this contraction, the electron experiences a *uniform* expansion (or contraction) of amount $G(a, \sigma)\lambda(\beta)$; where $\beta \equiv (v/c)$, $v=$ velocity of the electron, $c =$ velocity of light, $\lambda(\beta)$ is a function of β, a is the radius of the electron, σ is an arbitrary constant (*e.g.*, the electronic charge), and $G(a, \sigma)$ is an arbitrary function of a and σ. Let it be required to find that function, $\lambda(\beta)$, of β which will give the Lorentzian masses for *all* velocities.

2. Let symbols without subscript refer to the moving system, and those with subscript $_1$ to the corresponding static system. Then

$$\begin{aligned} x &= G(a, \sigma)\lambda(\beta)S^{1/2}\cdot x_1, \\ y &= G(a, \sigma)\lambda(\beta)y_1, \\ z &= G(a, \sigma)\lambda(\beta)z_1, \end{aligned} \tag{1}$$

where $S \equiv 1 - \beta^2$.

We also have the relation:[1]

$$\frac{\partial}{\partial t} = -v\frac{\partial}{\partial x}. \tag{2}$$

[1] O. W. Richardson, The Electron Theory of Matter, second edition (1916), p. 218. The electron is supposed to move in the direction of the X-axis. For the sake of completeness I have included some equations which are not necessary for the argument.

From equations (1) we get immediately:

$$\begin{aligned}\frac{\partial}{\partial x} &= \frac{1}{G\cdot\lambda(\beta)S^{1/2}}\cdot\frac{\partial}{\partial x_1},\\ \frac{\partial}{\partial y} &= \frac{1}{G\cdot\lambda(\beta)}\cdot\frac{\partial}{\partial y_1},\\ \frac{\partial}{\partial z} &= \frac{1}{G\cdot\lambda(\beta)}\cdot\frac{\partial}{\partial z_1},\end{aligned} \tag{3}$$

where $G \equiv G(a, \sigma)$. The equations which we will make use of are:

$$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\rho, \tag{4}$$

$$\nabla^2 \boldsymbol{U} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{U}}{\partial t^2} = -\rho\frac{\boldsymbol{v}}{c}, \tag{5}$$

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{U}}{\partial t} - \nabla\varphi, \tag{6}$$

and

$$\boldsymbol{H} = \nabla \times \boldsymbol{U}. \tag{7}$$

If we make use of equation (2), equation (4) may be put under the form:

$$S\left(\frac{\partial^2\varphi}{\partial x^2}\right) + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = -\rho. \tag{8}$$

Let us now change (8) to "static coördinates." From equations (3), it is easily seen that (8) becomes

$$\frac{1}{G^2\lambda^2(\beta)}\cdot\frac{\partial^2\varphi}{\partial x_1^2} + \frac{1}{G^2\lambda^2(\beta)}\frac{\partial^2\varphi}{\partial y_1^2} + \frac{1}{G^2\lambda^2(\beta)}\frac{\partial^2\varphi}{\partial z_1^2} = -\rho, \tag{9}$$

that is,

$$\frac{1}{G^2\lambda^2(\beta)}\left[\frac{\partial^2\varphi}{\partial x_1^2} + \frac{\partial^2\varphi}{\partial y_1^2} + \frac{\partial^2\varphi}{\partial z_1^2}\right] = -\rho. \tag{10}$$

The invariance of the electric charge gives:

$$\rho_1 = S^{1/2}G^3\cdot\lambda^3(\beta)\cdot\rho. \tag{11}$$

Equation (10) may now be written as

$$\frac{\partial^2\varphi}{\partial x_1^2} + \frac{\partial^2\varphi}{\partial y_1^2} + \frac{\partial^2\varphi}{\partial z_1^2} = -\frac{\rho_1}{S^{1/2}G\cdot\lambda(\beta)}. \tag{12}$$

But in the static system φ_1 satisfies the relation

$$\frac{\partial^2\varphi_1}{\partial x_1^2} + \frac{\partial^2\varphi_1}{\partial y_1^2} + \frac{\partial^2\varphi_1}{\partial z_1^2} = -\rho_1. \tag{13}$$

Hence, from equations (12) and (13), we conclude that

$$\varphi = \frac{1}{S^{1/2}\cdot G\cdot\lambda(\beta)}\cdot\varphi_1. \tag{14}$$

From equations (4) and (5) it is seen that

$$U = \beta\varphi. \tag{15}$$

3. From equations (6), (7), and (15), remembering that

$$U_y = U_z = 0; \quad U_x = \beta\varphi,$$

we get the following relations:

$$\begin{aligned} E_x &= -\frac{1}{G^2\cdot\lambda^2(\beta)}\cdot\frac{\partial\varphi_1}{\partial x_1}, \\ E_y &= -\frac{1}{S^{1/2}\cdot G^2\cdot\lambda^2(\beta)}\cdot\frac{\partial\varphi_1}{\partial y_1}, \\ E_z &= -\frac{1}{S^{1/2}\cdot G^2\cdot\lambda^2(\beta)}\cdot\frac{\partial\varphi_1}{\partial z_1}, \end{aligned} \tag{16}$$

and

$$\begin{aligned} H_x &= 0, \\ H_y &= \frac{\beta}{S^{1/2}\cdot G^2\cdot\lambda^2(\beta)}\cdot\frac{\partial\varphi_1}{\partial z_1}, \\ H_z &= -\frac{\beta}{S^{1/2}\cdot G^2\cdot\lambda^2(\beta)}\cdot\frac{\partial\varphi_1}{\partial y_1}. \end{aligned} \tag{17}$$

The momentum per unit volume is

$$\frac{1}{c}\,\mathbf{E}\times\mathbf{H}. \tag{18}$$

Therefore, the components of the momentum per unit volume are:

$$\begin{aligned} g_x &= \frac{\beta}{cSG^4\lambda^4(\beta)}\left[\left(\frac{\partial\varphi_1}{\partial y_1}\right)^2+\left(\frac{\partial\varphi_1}{\partial z_1}\right)^2\right], \\ g_y &= \frac{-\beta}{cS^{1/2}\cdot G^4\lambda^4(\beta)}\left[\frac{\partial\varphi_1}{\partial x_1}\cdot\frac{\partial\varphi_1}{\partial y_1}\right], \\ g_z &= -\frac{\beta}{S^{1/2}\cdot c\cdot G^4\lambda^4(\beta)}\left[\frac{\partial\varphi_1}{\partial x_1}\cdot\frac{\partial\varphi_1}{\partial z_1}\right], \end{aligned} \tag{19}$$

with

$$dxdydz = d\tau = \sqrt{S}\cdot G^3\cdot\lambda^3(\beta)d\tau_1 = \sqrt{S}\cdot G^3\cdot\lambda^3(\beta)dx_1dy_1dz_1.$$

Hence, considering only the X-component of the total momentum, we have

$$S_x = \frac{\beta}{c\cdot G\cdot\lambda(\beta)\sqrt{1-\beta^2}}\iiint\left[\left(\frac{\partial\varphi_1}{\partial y_1}\right)^2+\left(\frac{\partial\varphi_1}{\partial z_1}\right)^2\right]d\tau_1. \tag{20}$$

The value of

$$\iiint\left[\left(\frac{\partial\varphi_1}{\partial y_1}\right)^2+\left(\frac{\partial\varphi_1}{\partial z_1}\right)^2\right]d\tau_1$$

is known[1] to be $(e^2/6\pi a)$. Therefore,

$$S_x = \frac{\beta \cdot e^2}{6\pi ac \cdot G \cdot \lambda(\beta)\sqrt{1-\beta^2}}. \tag{21}$$

4. *To Find the Longitudinal and Transverse Masses.*—If F_x be the force in the direction of motion, m_L the longitudinal mass, and m_t the transverse mass, we have

$$F_x = \frac{\partial S_x}{\partial t}, \text{ by definition;}$$

$$= \frac{e^2\ddot{x}_1}{6\pi ac^2\sqrt{1-\beta^2}}\left[\frac{1}{G \cdot S \cdot \lambda(\beta)} - \frac{\beta\lambda'(\beta)}{G \cdot \lambda^2(\beta)}\right],$$

by substitution from (21). The longitudinal mass, however, is defined by

$$F_x = m_L\ddot{x}_1.$$

Therefore,

$$m_L = \frac{e^2}{6\pi ac^2\sqrt{1-\beta^2}}\left[\frac{1}{G \cdot S \cdot \lambda(\beta)} - \frac{\beta \cdot \lambda'(\beta)}{G \cdot \lambda^2(\beta)}\right]. \tag{22}$$

The transverse mass is given by

$$m_t = \frac{S_x}{v} = \frac{S_x}{\beta c}$$

$$= \frac{e^2}{6\pi ac^2 \cdot G \cdot \lambda(\beta)\sqrt{1-\beta^2}}, \tag{23}$$

But now, the expressions for the masses given by Lorentz are:

$$M_L = \frac{e^2}{6\pi ac^2(1-\beta^2)^{3/2}}, \tag{24}$$

$$M_t = \frac{e^2}{6\pi ac^2\sqrt{1-\beta^2}}. \tag{25}$$

Let us now determine $\lambda(\beta)$ so that we have always

$$M_L = m_L.$$

From equations (22) and (24) we get the following relation from which we are to determine $\lambda(\beta)$:

$$G \cdot \lambda^2(\beta) = \lambda(\beta) - \beta(1-\beta^2)\lambda'(\beta). \tag{26}$$

Let $\xi \equiv \lambda(\beta)$. Then,

$$G\xi^2 = \xi - \beta(1-\beta^2)\frac{d\xi}{d\beta};$$

[1] O. W. Richardson, The Electron Theory of Matter, second edition, page 233.

that is,

$$\frac{d\xi}{\xi(1-G\xi)} = \frac{d\beta}{\beta(1-\beta^2)},$$

and

$$\int \frac{d\xi}{\xi(1-G\xi)} + h = \int \frac{d\beta}{\beta(1-\beta^2)} + k,$$

or

$$\log\left[\frac{1-G\xi}{\xi}\right]\cdot\left[\frac{\beta}{\sqrt{1-\beta^2}}\right] = h - k = \text{const.} = \gamma.$$

Therefore,

$$\left[\frac{1-G\xi}{\xi}\right]\left[\frac{\beta}{\sqrt{1-\beta_2}}\right] = e^{\gamma} = \text{const.} = \delta;$$

that is,

$$\xi(G\beta + \delta\sqrt{1-\beta^2}) = \beta;$$

whence

$$\xi \equiv \lambda(\beta) = \frac{\beta}{G\beta + \delta\sqrt{1-\beta^2}}. \tag{27}$$

where G and δ are arbitrary constants independent of β.

Whatever G and δ may be, it will always be true that

$$M_L = m_L.$$

It follows, then, that as far as the *longitudinal* mass is concerned, the electron may expand to any extent whatever, and yet, the relation $M_L = m_L$ will always be true. If, however, we are to have the additional equality between M_t and m_t, that is, if we are also to have the relation

$$M_t = m_t$$

true for *all* velocities, then $G\cdot\lambda(\beta)$ must necessarily be equal to 1; that is, *there can be no uniform dilatation of the electron*. It is clear that this does not mean that a uniform expansion may not *actually* take place. What has been proved is that, if we *accept* Lorentz's formulæ for the transverse and longitudinal masses to be the *correct ones*, then *there can be no uniform dilatation* of the electron if the longitudinal and transverse masses given by (22) and (23) respectively are to be identical with those of Lorentz's formulæ for *all velocities*.

I am glad to tender my thanks to Professor G. F. W. Swann for the interest shown in the results herein described.

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MAXWELL'S LAW OF DISTRIBUTION OF VELOCITIES AND THE PRINCIPLE OF PROJECTIVE COVARIANCE.

BY I. MAIZLISH.

SYNOPSIS.

Derivation of Maxwell's Law of Distribution of Velocities with the Aid of the Principle of Projective Covariance.—In the ordinary derivation of this law there occurs a functional equation of the form: $g(x_1)g(x_2)g(x_3)g(x_4) = \psi(x_1^2 + x_2^2 + x_3^2 + x_4^2)$, whose solution is usually obtained on the supposition that the functions are differentiable. In this paper the solution is obtained without making use of the assumption, by applying the author's principle of projective covariance.

1. In the derivation of Maxwell's law of the distribution of velocities among a large number of molecules there occurs a certain functional equation whose solution is usually obtained by differentiation. The assumptions on which the solution is based are:

I. The functions occurring in the functional equation are continuous, single-valued, and real;
II. The independent variables are real;
III. The functions are differentiable.

2. Now it is known that a function may be continuous and yet not have a derivative. It is therefore desirable to inquire whether the functional equation can be solved without making use of assumption III. The object of this paper is to indicate a method of doing this by making use of the Principle of Projective Covariance. The problem will be generalized somewhat in that we shall consider it as a problem in a 4-dimensional space (S_4).

3. Consider, then, an S_4 and let x_1, x_2, x_3, x_4 be the rectangular coördinates of a point in this space. If we are dealing with a gas containing a large number (N) of molecules per unit volume, the *position* of any molecule will be known as soon as we know the values of x_1, x_2, x_3, x_4 appropriate to this molecule. Each molecule may therefore be represented by a single point in S_4. The points which are occupied by molecules will be called *molecular-points*. Let us now find the number of molecular-points for which

$$x_i \text{ lies between } x_i \text{ and } x_i + dx_i \ (i = 1, 2, 3, 4). \qquad (R)$$

4. Let $Ng(x_i)dx_i$ be the number of molecular-points for which x_i lies between x_i and $x_i + dx_i$; $i = 1, 2, 3, 4$. The same function, g, is used

for $i = 1, 2, 3, 4$, on account of the isotropy of S_4. If we now *assume* that x_1, x_2, x_3, x_4 are independent of each other, the number of molecular-points for which x_1, x_2, x_3, x_4 lie within the range (R) is

$$\Delta_4 N dx_1 dx_2 dx_3 dx_4 = N g(x_1) g(x_2) g(x_3) g(x_4) dx_1 dx_2 dx_3 dx_4; \qquad (1)$$

and, therefore, the number per unit volume is

$$\Delta_4 N = N g(x_1) g(x_2) g(x_3) g(x_4). \qquad (2)$$

The rectangular axes x_1, x_2, x_3, x_4 are, however, entirely arbitrary. Equation (2) must therefore be a function of the radius vector only; that is, we must have

$$g(x_1) g(x_2) g(x_3) g(x_4) = \psi(x_1^2 + x_2^2 + x_3^2 + x_4^2). \qquad (A)$$

This is the functional equation the solution of which will be obtained without making use of assumption III. We shall restrict our interest solely to the continuous solution of (A).

5. In order to see more clearly how equation (A) follows from the assumptions made, we will prove the following

THEOREM: *A necessary and sufficient condition in order that a function $F(x_1, x_2, x_3, x_4)$ be invariant under any orthogonal transformation is that it be a function of the radius vector only; that is, that*

$$F(x_1, x_2, x_3, x_4) = \psi(x_1^2 + x_2^2 + x_3^2 + x_4^2).$$

It will then follow that a necessary and sufficient condition that

$$g(x_1) g(x_2) g(x_3) g(x_4)$$

be invariant under any such transformation is that it be a function of the radius vector only.

PROOF: (a) The condition is sufficient. For let y_1, y_2, y_3, y_4 be the new variables, and

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}$$

the matrix of the transformation. By hypothesis,

$$\begin{aligned} F(x_1, x_2, x_3, x_4) &= \psi(x_1^2 + x_2^2 + x_3^2 + x_4^2), \\ &= \psi(y_1^2 + y_2^2 + y_3^2 + y_4^2), \end{aligned}$$

because an orthogonal transformation is defined as a linear transformation such that

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 \equiv y_1^2 + y_2^2 + y_3^2 + y_4^2.$$

By substitution,

$$\psi(y_1^2 + y_2^2 + y_3^2 + y_4^2) = F(y_1, y_2, y_3, y_4);$$

and

$$F(x_1, x_2, x_3, x_4) \equiv F(y_1, y_2, y_3, y_4).$$

(*b*) The condition is necessary. For we have

$$F(x_1, x_2, x_3, x_4) \equiv F(y_1, y_2, y_3, y_4)$$
$$\equiv F\left(\sum_{k=1}^{4} a_{1k}x_k, \sum_{k=1}^{4} a_{2k}x_k, \sum_{k=1}^{4} a_{3k}x_k, \sum_{k=1}^{4} a_{4k}x_k\right).$$

In the last equation place $x_2 = x_3 = x_4 = 0$. The result is

$$F(x_1, 0, 0, 0) \equiv F(a_{11}x_1, a_{21}x_1, a_{31}x_1, a_{41}x_1).$$

Thus, F must be of such a form as to be independent of a_{11}, a_{21}, a_{31}, and a_{41}. Since, however, a_{11}, a_{21}, a_{31}, and a_{41} cannot all be equal to 1, and, moreover, since the only condition existing between them is

$$a_{11}^2 + a_{21}^2 + a_{31}^2 + a_{41}^2 = 1,$$

we see that this must be the form in which they enter in F. This will in general be the case if and only if

$$F(a_{11}x_1, a_{21}x_1, a_{31}x_1, a_{41}x_1) \equiv \psi[(a_{11}x_1)^2 + (a_{21}x_1)^2 + (a_{31}x_1)^2 + (a_{41}x_1)^2].$$

This is to be true for all values of $a_{11}x_1$, $a_{21}x_1$, $a_{31}x_1$ and $a_{41}x_1$. Hence, if we replace $a_{k1}x_1$ by

$$a_{k1}x_1 + a_{k2}x_2 + a_{k3}x_3 + a_{k4}x_4, \quad k = 1, 2, 3, 4;$$

the result is that

$$F\left(\sum_{k=1}^{4} a_{1k}x_k, \sum_{k=1}^{4} a_{2k}x_k, \sum_{k=1}^{4} a_{3k}x_k, \sum_{k=1}^{4} a_{4k}x_k\right)$$
$$= \psi\left[\left(\sum_{k=1}^{4} a_{1k}x_k\right)^2 + \left(\sum_{k=1}^{4} a_{2k}x_k\right)^2 + \left(\sum_{k=1}^{4} a_{3k}x_k\right)^2 + \left(\sum_{k=1}^{4} a_{4k}x_k\right)^2\right];$$

that is,

$$F(x_1, x_2, x_3, x_4) \equiv F(y_1, y_2, y_3, y_4)$$
$$= \psi(y_1^2 + y_2^2 + y_3^2 + y_4^2).$$

But, in virtue of the relations existing between the coefficients of the matrix (M), it follows immediately that

$$\psi(y_1^2 + y_2^2 + y_3^2 + y_4^2) \equiv \psi(x_1^2 + x_2^2 + x_3^2 + x_4^2).$$

Therefore,

$$F(x_1, x_2, x_3, x_4) = \psi(x_1^2 + x_2^2 + x_3^2 + x_4^2).$$

The theorem is therefore proved.

6. The Principle of Projective Covariance[1] enables us to obtain the solution of the functional equation (A) with ease. This Principle asserts that the equations occurring in Physics are covariant under the transformation:

$$\begin{aligned} \xi_1 &= G(\sigma, r)x_1, \\ \xi_2 &= G(\sigma, r)x_2, \\ \xi_3 &= G(\sigma, r)x_3, \\ \xi_4 &= G(\sigma, r)x_4, \end{aligned} \tag{3}$$

where G is an arbitrary function of the distance (r) of any point from the origin of a system of coördinates in S_4, and an arbitrary constant σ; and x_1, x_2, x_3, x_4 are the Minkowski variables. To fix our ideas, consider the following special form of (3):

$$\begin{aligned} \xi_1 &= n^{\sigma - r^2} x_1, \\ \xi_2 &= n^{\sigma - r^2} x_2, \\ \xi_3 &= n^{\sigma - r^2} x_3, \\ \xi_4 &= n^{\sigma - r^2} x_4, \end{aligned} \tag{4}$$

where n is any number greater than 1. We have

$$g(x_1)g(x_2)g(x_3)g(x_4) = \psi(x_1^2 + x_2^2 + x_3^2 + x_4^2). \tag{A}$$

By the Principle of Projective Covariance, we also have

$$g(\xi_1)g(\xi_2)g(\xi_3)g(\xi_4) = \psi(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2). \tag{B}$$

In equation (A) set $x_2 = x_3 = x_4 = 0$, and in equation (B) set $\xi_2 = \xi_3 = \xi_4 = 0$. The result is

$$g^3(0)g(x_1) = \psi(x_1^2),$$

and

$$g^3(0)g(\xi_1) = \psi(\xi_1^2).$$

Hence, since, from physical considerations, $g(0)$ exists,

$$\frac{\psi(x_1^2)}{g(x_1)} = \frac{\psi(\xi_1^2)}{g(\xi_1)}. \tag{5}$$

7. Now ξ_1 may be looked upon as an independent variable. For, despite the fact that x_1 and ξ_1 are connected by the transformation (4), the value of ξ_1 is not determined by x_1 alone: it also depends on x_2, x_3, and x_4. But then, the left-hand side of equation (5) is a function of x_1 only, and the right-hand side is a function of ξ_1 only. Hence, equation (5) can only be true if each member of this equation is equal to a constant. Therefore,

$$\frac{\psi(x_1^2)}{g(x_1)} = \text{const.} = \beta^3, \text{ say.} \tag{6}$$

[1] I. Maizlish, PHYSICAL REV., July, 1921.

8. In equation (A) set $x_2 = x_3 = x_4 = 0$. The result is

$$g(x_1)g^3(0) = \psi(x_1^2);$$

from which, by equation (6), we get

$$g^3(0)\frac{\psi(x_1^2)}{\beta^3} = \psi(x_1^2);$$

that is,

$$g(0) = \beta. \tag{7}$$

Moreover, from equation (A) it is easily seen that

$$g(0)g(0)g(x_1)g(x_2) = \psi(x_1^2 + x_2^2),$$

that is,

$$\beta^2 g(x_1)g(x_2) = \psi(x_1^2 + x_2^2). \tag{8}$$

Multiply each member of equation (8) by β^4. Thus,

$$\beta^3 g(x_1)\beta^3 g(x_2) = \beta^4\psi(x_1^2 + x_2^2),$$

that is,

$$\psi(x_1^2)\psi(x_2^2) = \beta^4\psi(x_1^2 + x_2^2) \tag{9}$$

by substitution from (6). Now divide each member of (9) by β^8.

The result is

$$\frac{\psi(x_1^2)}{\beta^4} \cdot \frac{\psi(x_2^2)}{\beta^4} = \frac{\psi(x_1^2 + x_2^2)}{\beta^4};$$

and, on placing

$$\varphi(x) \equiv \frac{\psi(x)}{\beta^4}, \tag{9'}$$

this becomes

$$\varphi(x_1^2)\varphi(x_2^2) = \varphi(x_1^2 + x_2^2). \tag{10}$$

The last equation may be written

$$\varphi(x)\varphi(y) = \varphi(x + y), \tag{11}$$

by placing $x = x_1^2$, and $y = x_2^2$.

9. Now the functional equation (11) has been solved by Cauchy,[1] Darboux,[2] and others. Assuming that φ is continuous, single-valued, and real; and that x and y are real, Cauchy has shown that the *only* (continuous) solution of (11) is

$$\varphi(x) = a^x,$$

where a is an arbitrary constant. But these assumptions are identical with those we made at the beginning. Hence, from eqs. (10) and (11) we get

$$\varphi(x_1^2) = a^{x_1^2}.$$

[1] Cauchy, Œuvres (2), 3 (1897), p. 98.

[2] Darboux, Math. Ann., Bd. 17, 1880, p. 56.

In view of eqs. (6) and (9[1]) we now have

$$a^{x_1^2} = \varphi(x_1^2) = \frac{\psi(x_1^2)}{\beta^4} = \frac{\beta^3 g(x_1)}{\beta^4};$$

$$\therefore \quad g(x_1) = \beta a^{x_1^2}. \tag{12}$$

Thus, in general,

$$g(x_i) = \beta a^{x_i^2}, \quad (i = 1, 2, 3, 4). \tag{13}$$

10. It follows that the number of molecular-points for which x_1, x_2, x_3, x_4 lie within the range (R) is, by (1) and (13),

$$\Delta_4 N dx_1 dx_2 dx_3 dx_4 = N\beta^4 a^{[x_1^2+x_2^2+x_3^2+x_4^2]} dx_1 dx_2 dx_3 dx_4;$$

and the number per unit volume is therefore:

$$\Delta_4 N = N\beta^4 a^{[x_1^2+x_2^2+x_3^2+x_4^2]}. \tag{14}$$

The constant a is arbitrary: we may therefore put it equal to e^{-h}, where e is the Napierean base, h is a constant, and the minus sign is prefixed to the h for otherwise $\Delta_4 N$ would increase indefinitely with $(x_1^2 + x_2^2 + x_3^2 + x_4^2)$. Thus, equation (14) may be written as

$$\Delta_4 N = N\beta^4 e^{-h[x_1^2+x_2^2+x_3^2+x_4^2]}. \tag{15}$$

11. The solution obtained is a generalization of Maxwell's. By putting $x_4 = 0$, and remembering that $g(0) = \beta$, equation (15) reduces to

$$\Delta_3 N = N\beta^3 e^{-h[x_1^2+x_2^2+x_3^2]} \tag{16}$$

12. We may regard x_1, x_2, x_3 as any set of rectangular axes in a space of three dimensions (S_3). Take these axes as the components of a velocity c defined by

$$c^2 = u^2 + v^2 + w^2;$$

where

$$x_1 \equiv u,$$

$$x_2 \equiv v,$$

and

$$x_3 \equiv w.$$

Making this substitution, (16) becomes

$$\Delta_3 N = N\beta^3 e^{-h[u^2+v^2+w^2]}.$$

13. The number of molecules for which u, v, w lie within the range u and $u + du$, v and $v + dv$, w and $w + dw$, respectively, is now seen to be

$$\Delta_3 N du dv dw = N\beta^3 e^{-h[u^2+v^2+w^2]} du dv dw,$$

which is the solution given[1] by Maxwell.

[1] Maxwell, Collected Papers, Vol. I., p. 381.

14. Thus, provided we justify every step in the process of arriving at equation (A), and restrict our interest to the continuous solution of this equation, under the specified conditions, the solution (solutions) depends (depend) on that (those) of

$$\varphi(x)\varphi(y) = \varphi(x + y). \qquad (C)$$

But under the same restrictions as those specified at the beginning of this paper, there is *only one* solution of (C). Hence, there is only *one* solution of (A), and, by the application of the Principle of Projective Covariance, this has been shown to be

$$g(x) = \beta e^{-\lambda x^2}.$$

I am grateful to Professor R. W. Brink, of the Department of Mathematics, for examining the manuscript.

DEPT. OF PHYSICS,
THE UNIVERSITY OF MINNESOTA,
MINNEAPOLIS,
March 23, 1922.

A MODIFICATION OF VAN DER WAALS' EQUATION.

BY W. P. BOYNTON AND ARTHUR BRAMLEY.

SYNOPSIS.

The proposed equation is $(p + a/v^2)(v - b) = RT/(1 + \psi^2/T^2)$, where ψ is a characteristic temperature. It represents the behavior of CO_2 and other substances more closely and gives better values of the ratio T_c/p_cv_c. While the corrective factor is purely arbitrary, it resembles those of certain theoretical equations having a rational basis and also lends itself to mathematical treatment. It is shown that not only the energy but also the specific heat and entropy derived from this equation approach zero values at absolute zero in agreement with conclusions from the researches of Nernst.

Theoretical formulæ for specific heat (c_v), *internal energy and entropy* as functions of T and v, are derived from thermodynamic laws and the equation of state given above.

I. INTRODUCTION.

THE authors have attempted to derive an equation of state which shall be more satisfactory than those previously proposed. To be of value such an equation must describe the behavior of actual substances to some degree of approximation, and must be mathematically tractable. If it fails in the first point, it is worthless; if in the second, useless. It is more satisfying if it also has a rational basis.

An excellent example of such an equation is the familiar formulation of the Boyle-Charles laws,

$$pv = RT,$$

which has a rational basis in the kinetic theory of gases, is a very useful first approximation, and is exceedingly tractable. It is confessedly only a first approximation, describing merely the "ideal" (or perfect) gas. A typical second approximation is the famous equation of van der Waals,

$$(p + a/v^2)(v - b) = RT,$$

which also has a rational basis, is quite fairly tractable, and corresponds qualitatively to the behavior of substances capable of existing in both the gaseous and liquid forms, including such phenomena as the critical state, and by the aid of thermodynamical considerations, the coexistence and equilibrium of the two states.

II. THERMODYNAMIC RELATIONS.

A satisfactory equation of state must be consistent with the fundamental laws of thermodynamics, and conversely, the laws of thermody-

namics are sufficient to determine the forms of the equations for specific heat, entropy, internal energy and other thermodynamic properties, provided an equation of state is employed which gives a sufficiently close approximation to the facts of the behavior of substances.

The familiar formulation of the first and second laws, on the assumption that the only work is that against external pressure, is

$$TdS = dU + pdv,$$

where

S = entropy,
U = internal energy.

This may be expanded to the form

$$TdS = \frac{\partial U}{\partial T}dT + \left(\frac{\partial U}{\partial v} + p\right)dv.$$

The ordinary transformations lead to the relations

$$\frac{\partial S}{\partial T} = \frac{1}{T}\frac{\partial U}{\partial T}; \quad \frac{\partial S}{\partial v} = \frac{1}{T}\left(\frac{\partial U}{\partial v} + p\right); \quad \frac{\partial U}{\partial v} = T\frac{\partial p}{\partial T} - p; \quad \frac{\partial S}{\partial v} = \frac{\partial p}{\partial T}.$$

Discarding the usual easy but unnecessary assumption that the specific heat is a constant, independent of temperature, its value may be found as follows. Differentiating partially by T the value of $\partial U/\partial v$,

$$\frac{\partial^2 U}{\partial T \partial v} = \frac{\partial p}{\partial T} + T\frac{\partial^2 p}{\partial T^2} - \frac{\partial p}{\partial T} = T\frac{\partial^2 p}{\partial T^2},$$

and integrating with respect to v,

$$\frac{\partial U}{\partial T} = c_v = T\int \frac{\partial^2 p}{\partial T^2}dv + f(T).$$

The ordinary assumptions that p is a linear function of T and that c_v is a constant evidently constitute a special case of this general relation. From this value of c_v and the relations given above as existing between the various partial derivatives it is possible to deduce values for U and S.

$$U = \int\left[\left(T\int\frac{\partial^2 p}{\partial T^2}dv\right)dT + \left(T\frac{\partial p}{\partial T} - p\right)dv\right] + \int f(T)\,dT,$$

$$S = \int\left[\left(\int\frac{\partial^2 p}{\partial T^2}dv\right)dT + \frac{\partial p}{\partial T}dv\right] + \int\frac{f(T)}{T}dT.$$

III. Proposed Equation of State.

The researches of Nernst at low temperatures have led to the formulation of his "heat theorem" which states that the temperature coeffi-

cient of any property of a body, which varies with the temperature and approaches a finite limiting value as the temperature is lowered, is zero. This theorem leads to the conclusion that the specific heat of all substances approaches zero as the temperature is lowered, a result which is verified by experiment.

This theorem has also lead Planck and others to state that the entropy approaches the limiting value zero at the absolute zero.

These two conditions, namely the zero values of c_v and of S, together with the obvious fact that the internal energy, U, must also vanish at the absolute zero of temperature, determine completely the boundary conditions at the lower limit. An inspection of the generalized thermodynamic equations obtained above shows that the condition that S shall vanish at the absolute zero can only be satisfied if p is such a function of T that $\partial^2 p/\partial T^2$ does not vanish identically.

The assumptions of the kinetic theory that the pressure and the internal energy of a gas are both associated with the kinetic energy of the rectilinear motion of its molecules lead to the suggestion that c_v and the gas constant R may be at least similarly related to the temperature. Attempts have been made to find a rational formula for c_v as a function of the temperature. Of these two at least have been fairly successful, that of Debye based upon the quantum hypothesis, and the other, of A. H. Compton[1] based upon the notion of agglomeration of molecules. Curves plotted from these two equations agree fairly closely with each other as well as with the experimental data, but both are transcendental in form and hence do not lend themselves readily to mathematical manipulation. For that reason we are suggesting the use of the algebraic form

$$\frac{k^2T^2}{1 + k^2T^2},$$

in which k is the reciprocal of a temperature. Comparing this with Compton's form

$$\frac{C}{C_\infty} = e^{-\tau/T}\left(\frac{\tau}{T} + 1\right)$$

for the particular value $k = 5/3\tau$, the resulting curves are in substantial agreement for all values of T/τ above 0.5, within that range differing by amounts varying from 0.001 to 0.007; but below that point the difference is greater, the algebraic form exceeding the other at the most by about 0.06. Nevertheless the simpler algebraic curve is of the same general form, and may be regarded as a permissible approximation to the more exact but also more complicated rational form, when the main

[1] PHYSICAL REVIEW (2), 6, pp. 377–389, 1915.

purpose is a general qualitative description only. Following this suggestion we assume that

$$R = R_\infty \frac{T^2/\psi^2}{1 + T^2/\psi} = R_\infty/(1 + \psi^2/T^2),$$

where ψ is a characteristic temperature, and write a modified form of van der Waals' equation

$$(p + a/v^2)\,(v - b) = R^\infty T/(1 + \psi^2/T^2).$$

From this equation the critical data can be derived without difficulty and the ordinary form of the equation in reduced coördinates obtained, except that θ, the "reduced" temperature, will be a somewhat complicated function of T, T_c and ψ. Substituting from this modified equation of state the partial derivatives of p in the thermodynamically derived equations for c_v, U and S, we obtain, dropping the subscript of R,

$$c_v = 2R\frac{3\psi^4/T^4 - \psi^2/T^2}{(1 + \psi^2/T^2)^3} ln\,(v/b - 1) + f(T),$$

$$U = 2RT\frac{\psi^2/T^2}{(1 + \psi^2/T^2)^2} ln\,(v/b - 1) - a/v + a/b + \int_0^T f(T)dT,$$

$$S = R\frac{3\psi^4/T^4 + \psi^2/T^2}{(1 + \psi^2/T^2)^2} ln\,(v/b - 1) + \int_0^T \frac{f(T)}{T} dT.$$

These functions have been integrated with respect to v between the limits b and v, and since $v = b$ only when $T = O$, the product $T^2 ln(v/b - 1)$ and similar terms involving higher terms of T all vanish at the lower limit.

We shall assume that we may write

$$f(T) = \Sigma M \frac{k^2T^2}{1 + k^2T^2},$$

that is, as one term, or a sum of terms, of the form suggested. Since the mathematical behavior of the sum is similar to that of a single term, we may perform the integrations indicated, and write as a special case

$$f(T) = M\frac{k^2T^2}{1 + k^2T^2},$$

$$\int_0^T f(T)\,dT = M\int_0^T \frac{k^2T^2}{1 + k^2T^2}dT = M\left(T - \frac{1}{k}\tan^{-1}kT\right),$$

$$\int_0^T \frac{f(T)}{T}dT = M\int_0^T \frac{k^2T}{1 + k^2T^2}dt = \frac{M}{2} ln\,(1 + k^2T^2),$$

in which k is the reciprocal of a characteristic temperature. All these functions vanish for $T = O$, and hence the functions derived as repre-

senting c_v, U and S satisfy completely the boundary conditions at the lower limit.

IV. CONCLUSIONS.

We have proposed a modification of van der Waals' equation of the form

$$(p + a/v^2)(v - b) = RT/(1 + \psi^2/T^2),$$

which exhibits the following advantages:

1. For high temperatures it is practically identical with the familiar van der Waals equation, and like it, when volumes also are large, closely approximates the equation of the ideal gas.

2. Like the equation of van der Waals it exhibits the phenomena of the critical state, giving identical formulæ for the critical volume and pressure, but a more complicated one for the critical temperature.

3. The proposed equation gives numerical results which agree with the external behavior of actual substances apparently as closely as any previously suggested equations; *e.g.*, it fits Andrews' data on carbon dioxide fully as closely as does the equation of Clausius, which was developed expressly for that purpose.

4. A common test is the value of the ratio RT_c/P_cv_c which for van der Waals' equation has the value 8/3, while observed values as quoted range mainly from 3.7 to 5.0. This equation gives

$$\frac{RT_c}{p_cv_c} = \frac{8}{3}(1 + \psi^2/T^2_c),$$

which takes the value 3.71 for $T_c/\psi = 1.6$ or the value 5.0 for $T_c/\psi = 1.068$.

5. The value of the specified heat deduced from it shows the proper behavior at low temperatures, and especially in the neighborhood of the absolute zero. The formula deduced, involving as it does the volume v and the possibility that $f(T)$ may be a series of several terms, can be made to conform quite closely to the observed data, even for a substance like water, over a temperature range including one or both changes of state.

6. The expression for the entropy does not involve an infinite constant of integration, but on the contrary vanishes at zero, and has a finite value for finite volumes and temperatures.

Further consideration of the properties of this equation is reserved for future papers.

UNIVERSITY OF OREGON,
EUGENE, OREGON,
February 20, 1922.

THE EFFECT OF NASCENT HYDROGEN ON HARD STEEL MAGNETS.

By John Coulson.

Synopsis.

Aging Effect of Nascent Hydrogen on the Magnetic Moment of Hardened Steel Magnets.—When magnets, well seasoned by repeated heating and cooling, are exposed to the action of nascent hydrogen by being made cathodes in a sulfuric acid bath, the magnetic moment decreases, reaching in time a permanent value from 5 to 20 per cent. less than the original moment. The aging process is accelerated by raising the temperature of the electrolyte, the total time required being only one hour or less at 60° C. For *drill rod* (carbon 1.2 per cent.), quenched from 800° C., the change occurs gradually; but for *tungsten magnet steel* most of the decrease occurs suddenly after an exposure which, for the electrolyte at 60° C., varies from 3 to 15 minutes as the quenching temperature is increased from 700 to 850° C., the corresponding sudden changes in moment decreasing from 19 to 4 per cent. The behavior of *spring steel* (carbon 0.8 per cent.) is very similar, but the sudden changes occur later. After aging in this way, hammering the magnets has practically no effect.

Penetration of Nascent Hydrogen into Mild Steel.—Photographs of fractured cold-rolled rods show that in 40 seconds the hydrogen penetrated about 1 mm.

THE magnetic moment of *permanent* magnets changes quite perceptibly even following the most careful "aging"[1] after magnetization. This so-called aging effect goes on at a diminishing rate notwithstanding such magnets be kept isolated in a neutral field. The desirability of hastening the final condition, where the magnetic moment ceases to change, led to the present investigation.

Bar magnets of "Special Alloy Magnet Steel" and high carbon steels were treated electrolytically in a strongly acidulated electrolyte. It was found that the magnetic moment of magnets treated as cathode undergoes a marked diminution, and that the character of the change depends on the nature of the steel and on the hardening temperature.

Careful and elaborate investigations have shown that steel absorbs hydrogen and other gases, and the effect of such gases on the mechanical properties of the metal has been studied in detail.[2] Most of this work,

[1] Wiedemann, Pogg. Ann., 1852, 1858, 1864. Dufour, Pogg. Ann., 99, 1855; Arch. de Geneve, 1857. Favé, Comptes Rendus, 72, 1876. Gaugain, Comptes Rendus, 72, 1876. Trowbridge, Am. Jour. of Science, 1881. Kohlrausch, Wied. Ann., 1884. Barus and Strouhal, Bulletin of the U. S. Geological Survey, No. 14, 1885. Peirce, Am. Jour. of Science, 1896. Durward, Am. Jour. of Science, 1898. Peirce, Proc. Amer. Acad. Arts and Sciences, 1905.

[2] Bellock, Comptes Rendus, 149, 1909. Richards and Behr, Zeitschr. Phys. Chem., 58,

however, has been carried out on steels which were exposed to the gases at temperatures considerably above normal or room temperature. It is not generally accepted that steel absorbs hydrogen to any extent at normal temperatures, therefore, hydrogen is not considered to play here a vital part at ordinary temperatures in the structural or other changes taking place in the metal. Nevertheless, the author,[1] and others[2] have found that nascent hydrogen penetrates steel surprisingly rapidly at temperatures as low as 20° C.

The permeability of steel for hydrogen at these low temperatures suggested that the permanency of the magnetic moment of hardened steel magnets might well be influenced by the absorption of hydrogen. In the course of the investigation fifty or more bar magnets of length 14.4 cm. were carefully made and tested for changes in the magnetic moment after treatment. Most of the specimens were quenched to glass hardness from 850° C. Some were quenched at a lower temperature, about 700° C. It will be seen that the hardening temperature is of great significance in the changes taking place within the steel when hydrogen is absorbed.

Following the hardening process the specimens were magnetized to saturation inside a solenoid many times their length. After removal from the solenoid care was taken to keep the magnets isolated from all magnetic material until their history had been completed. The usual fall in the magnetic moment of the magnets, due to repeated heating and cooling, was observed. This operation was continued until evidence of further change of magnetic moment had disappeared. The heating operation was carried out with the magnet sealed in an air-tight glass tube immersed in boiling water.

The preliminary aging having been completed, the magnet under test was subjected to the action of nascent hydrogen by making it the cathode in an electrolytic cell, while the lead container served as anode. The electrolyte used was a 25 per cent. solution of H_2SO_4, and the current through the cell was kept at 2 amperes. The magnets were exposed to the action of the hydrogen for increasing time intervals, at the outset, usually 15 or 30 seconds. After each exposure the magnetic moment of the magnet was measured at room temperature.

1907. Hein, Zeitschr. Phys. Chem., 58, 1907. Goutal, Comptes Rendus, 148, 1909. Sieverts and Krumhaar, Berl. Deutsche Chem. Ges., 43, 1910. Rumpf, Akad. Wiss. Wien, Sitz. Ber., 119, 1910. Charpy, Bonnerot, Comptes Rendus, 152, 1911. Stead, Iron and Steel Industry, J., 103, 1921; Engineering, 111, 1921. Thompson and Richardson, Met. and Chem. Eng., 16, 1917. Smith, Am. Electro-Chem. Soc. J., 1918.

[1] Coulson, Am. Electro-Chem. J., 1917.

[2] Bellaty and Lussana, R. Istituto Veneto Atti., 1888; 1891. N. Cimento, 5, 1913. Charpy and Bonnerot, Comptes Rendus, 152, 1911; 154, 1912. Bellaty and Lussana, N. Cimento, 5, Ser. 6, 1913.

The measurements were made by placing on the opposite side of a magnetometer a standard bar magnet in Gauss's A position, held, and protected completely from rapid temperature changes, within a wooden carriage sliding on a horizontal scale. This compensating magnet, having been fixed with its center at a convenient distance d_0 from the center of the magnetometer needle, brought the needle back into the meridian when the center of the magnet under observation was at the fixed distance d. Let M_0 be the magnetic moment of the standard magnet and $2l_0$ its magnetic length, and let M_1 and M_1' be the moments of the magnet under test, before and after treatment respectively, and $2l$ its magnetic length. The needle will be deflected through the angle α so that

$$\frac{2M_0d_0}{(d_0^2 - l_0^2)^2} = \frac{2M_1d}{(d^2 - l^2)^2},$$

$$\frac{2M_1d}{(d^2 - l^2)^2} - \frac{2M_1'd}{(d^2 - l^2)^2} = \frac{2d(M_1 - M_1')}{(d^2 - l^2)^2} = H \tan \alpha,$$

$$\frac{M_1 - M_1'}{M_1} = \frac{(d_0^2 - l_0^2)^2}{2M_0 d_0} \cdot H \tan \alpha = \frac{\tan \alpha}{\tan \alpha_0},$$

where H is the horizontal component of the earth's magnetic field, and α_0 is the angle through which M_0 would deflect the needle if M_1 were absent.

The method of procedure was then as follows:

The standard magnet was placed with its center at a known distance from the magnetometer, and its carriage clamped firmly in position. The magnet to be tested was placed on the opposite side of the magnetometer and adjusted in such position as to bring the needle back into the meridian. Its holder was then clamped and the distance of the center of the magnet from the magnetometer was recorded. After subjecting the magnet to treatment, it was placed again in its holder and the deflection of the magnetometer needle observed with the magnet at room temperature. The dimensions and the magnetic moment of the standard magnet being known, the magnetic moment and the fractional loss of the moment of the magnet under test could be easily calculated from the above formulæ.

It may be of interest to say that the Special Magnet Steel was a tungsten alloy steel in the form of square bars 8.0 mm. cross section, and was received direct from the Crucible Steel Co., Pittsburgh. The spring steel and polished drill rod were in the form of rods 6.4 mm. in diameter, and were bought in the open market. The carbon content was about 0.80 and 1.20 respectively.

Typical results obtained with magnets of the above materials treated

electrolytically at 20° C. and 60° C. are shown, some in tabular form, others by curves. Table I. and Fig. 1 are results obtained for magnets

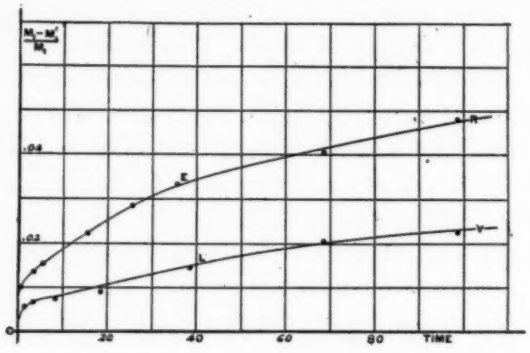

Fig. 1.

Shows the fractional change in the magnetic moment as a function of the time of exposure to the hydrogen at 20° C. Polished drill rod: Curve *OER*. Special alloy magnet steel: Curve *OLV*.

treated at 20° C. The fractional loss of magnetic moment and the corresponding time exposure of the magnet to the action of the hydrogen are given. The upper section of the table and the curve *OER* of the figure represent observations made on two magnets of drill rod, of which many were tested. For like treatment the results were in very good agreement. Representative data are given in the lower half of the table for a magnet of spring steel, and similar data for a magnet of special magnet steel are shown in curve form, Curve *OLV*, of the figure.

TABLE I.

History.	Exposure (Minutes).	$\frac{M_1 - M_1'}{M_1}$.	M.	Exposure (Minutes).	$\frac{M_1 - M_1'}{M_1}$.	M.
Drill Rod.	0		1836	20.0	.0245	1788
Quenched from 800° C.	1.0	.0120	1814	30.0	.0270	1786
Magnetic moment after	3.0	.0150	1809	40.0	.0302	1781
magnetization,	5.0	.0168	1805	50.0	.0320	1777
M = 1836.	10.0	.0198	1800	60.0	.0342	1773
Spring Steel.	0		1919	20.0	.0130	1894
Quenched at 850° C.	0.5	.0030	1913	30.0	.0142	1892
Magnetic moment after	1.5	.0046	1910	40.0	.0158	1889
magnetization,	3.0	.0070	1906	50.0	.0170	1886
M = 1919.	5.0	.0090	1902	60.0	.0175	1885
	10.0	.0111	1898	70.0	.0180	1884

Temperature of exposure: electrolyte kept at 20° C.

Magnets made from drill rod, treated electrolytically, show a greater

aging effect than magnets of the other steels. The spring steel magnets change more slowly at the outset than those of special magnet steel, but after 50 minutes' exposure, approximately, the fractional decrease of their respective moments had the same value. Usually after this stage the loss in magnetic moment for magnets of spring steel was greater than for the special magnet steel. From two to three hours' exposure to the electrolytic hydrogen decreased the magnetic moment of the magnet to a fixed value. Hammering the magnets had little or no effect in changing further their magnetic moment. Careful weighing before and after treatment showed that the magnets did not lose weight during exposure.

Numerous magnets of spring steel and special magnet steel, treated at 20° C., showed a sudden drop in magnetic moment before reaching a constant value. It occurred usually after an hour's treatment or more and was of a magnitude of one or two per cent. The change before and after being only a fraction of one per cent.

The rapidity with which the magnetic moment settles to a final value depends on the temperature of the electrolyte in which the magnets are brought under the action of the electrolytically evolved hydrogen. Moreover, the transition to this final state, due to the action of the hydrogen, is surprisingly different for the various specimens.

TABLE II.

History.	Exposure (Minutes).	$\frac{M_1 - M_1'}{M_1}$	M.	Exposure (Minutes).	$\frac{M_1 - M_1'}{M_1}$	M.
Drill Rod.	0		2040	30.0	.101	1834
Quenched from 800° C.	0.5	.0150	2010	40.0	.109	1816
Magnetic moment after	1.0	.0259	1987	50.0	.116	1804
repeated heating and	3.0	.0585	1921	60.0	.121	1793
cooling,	5.0	.0820	1873	80.0	.127	1781
M = 2040.	10.0	.0881	1860	90.0	.129	1777
	20.0	.095	1846			
Special Alloy Magnet	0		1697	10.0	.0129	1675
Steel.	0.5	.0009	1695	20.0	.0538	1650
Original magnet	2.0	.0097	1680	25.0	.0550	1603
moment, M = 1761.	5.0	.0115	1677	35.0	.0565	1601
Moment after repeated heating and cooling, M = 1697.						
After being dropped 1 ft. end on fifty times on a block of stone	.....	.....	.....	.....	.....	..1600

Temperature of exposure: electrolyte kept at 60° C.

Table II. and Fig. 2 show results obtained for magnets of different

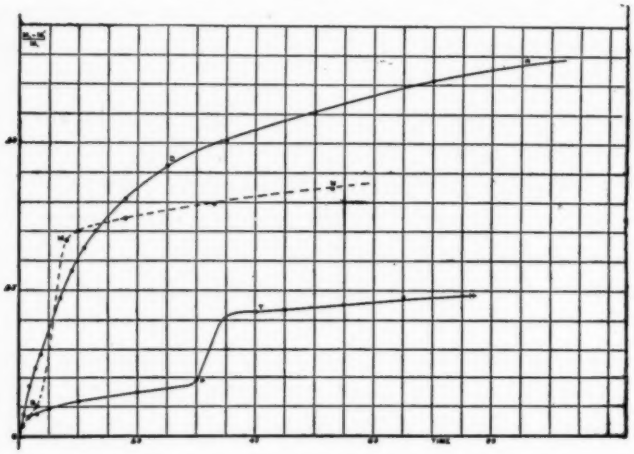

Fig. 2.

Shows the fractional change in the magnetic moment as a function of the time of exposure to the hydrogen at 60° C. Polished drill rod: Curve *ODR*. Special alloy magnet steel: Curve *OMN*. Spring steel: Curve *OPTH*.

material, treated at 60° C. The upper section of the table and the Curve *ODR* of the figure are for magnets made from polished drill rod, quenched at slightly different temperatures to glass hardness. As will be seen, the magnitude of the aging is not the same for the two magnets, but the character of the results is very similar. The final value of the magnetic moment is reached in approximately the same time of exposure. The decrease being about 14 per cent.

The broken Curve *OMN* of the figure and the results tabulated in the lower portion of the table illustrate the character of this aging process on magnets of special alloy magnet steel. The quenching temperature for these magnets differed by about 50° C. The specimen quenched at the lower temperature, 800° C., and indicated by the curve shows a very sudden change in magnetic moment after a few minutes' exposure to the hydrogen in the heated electrolyte. For the specimen quenched at the higher temperature, 850° C., this sudden fall in magnetic moment takes place after a longer exposure. This sudden change in moment is, also, very marked in the case of magnets made of spring steel, but it takes place usually after 20 or 30 minutes' exposure. The curve *OPTH* of the figure is a fair example of results obtained for specimens of this steel when quenched glass hard. Examination of the results will show that after this sudden transition took place the change in the magnetic moment was comparatively small, even after prolonged exposure. Hammering the magnets at this stage caused practically no alteration of the magnetic moment.

This precipitous change in the magnetic moment of magnets made from special magnet steel and from spring steel appears to be considerably magnified when the specimens are quenched at temperatures below 850° C. Table III. shows results obtained with a magnet of special magnet steel quenched at 700° C. These results are shown graphically in Fig. 3, curve *PRTE*. Examination will show that after 2 minutes'

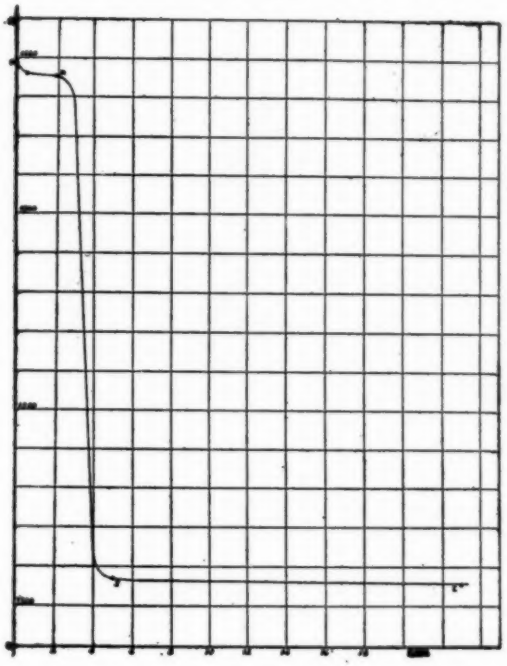

Fig. 3.

Shows the moment of a magnet of special alloy magnet steel quenched at 700° C., as a function of the time of exposure to the hydrogen. Temperature of the electrolyte 60° C.

TABLE III.

History.	Exposure (Minutes).	$\frac{M_1 - M'}{M_1}$.	M.
Quenched from 700° C.	0		1378
Magnetic moment after magnetization,	0.5	.0043	1372
$M = 1452$.	2.0	.0046	1371
Moment after repeated heating and cooling,	5.0	.1916	1114
$M = 1378$.	8.0	.1921	1113
	13.0	.1921	1113
	23.0	.1927	1112

Special Alloy Magnet Steel.

Temperature of exposure: electrolyte kept at 60° C.

exposure to the electrolytic hydrogen the magnetic moment of the magnet dropped more than 19 per cent., while up to this point the decrease in magnetic moment was only a small fraction of one per cent. After the sudden change occurred, further exposures to the hydrogen had practically no effect on the magnetic moment.

Fig. 4, which is taken from an earlier paper,[1] illustrates the rapidity

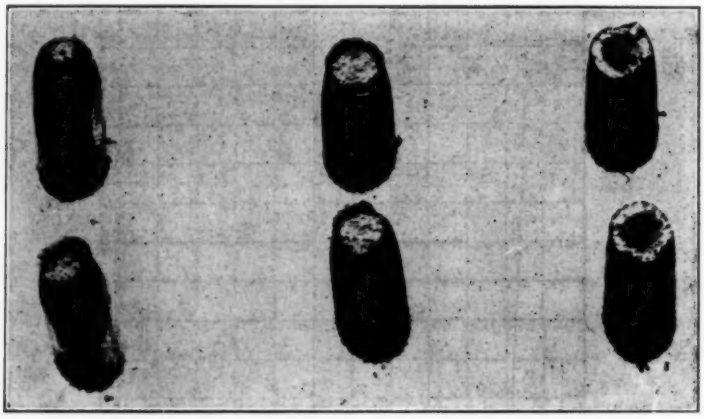

Fig. 4.

Shows the fracture of cold-rolled steel broken under stress. *A* and *B* without exposure to hydrogen. *C* exposed 40 seconds to electrolytic hydrogen. The bright band shows depth of penetration.

with which electrolytic hydrogen penetrates mild steel. *A*, *B*, and *C* are test specimens, 6 mm. in diameter, cut from the same rod of cold-rolled steel. *A* and *B* were broken under stress without exposure to hydrogen. *C*, on the other hand, was broken after an exposure of 40 seconds. The bright band of this test piece indicates the depth of penetration of the hydrogen during this interval.

An investigation which is in progress on a wide range of steels shows that both the magnetic permeability and the specific resistance decrease on exposure of the metal to the electrolytic hydrogen. It is hoped that when this work is reported, a complete explanation of the changes taking place within the metal will be given.

I wish to acknowledge the valuable assistance given me by Mr. W. O. Smith, who has helped me in all the work.

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[1] Coulson, Am. Electro-Chem. J., 1917.

IMPROVEMENT IN THE DETERMINATION OF THE RADIUM CONTENT OF LOW-GRADE RADIUM-BARIUM SALTS.

BY VICTOR F. HESS AND ELIZABETH E. DAMON.

(Read before the meeting of the American Physical Society, New York, Feb. 25, 1922.)

SYNOPSIS.

Improved Gamma-Ray Method of Determining the Radium Content of Low-grade Radium Salts.—Because of absorption of the rays by the salt, the present gamma-ray method is rather inaccurate for concentrations less than 10^{-6}, and for concentrations above 10^{-7} the emanation method is too sensitive. To bridge the gap, the authors have modified the first method by adopting a shallow container with two curved sides, each concentric with the cylindrical string electrometer used to measure the rays, and only a small fraction of the radius apart. This shape has the effect of making the mean absorption due to the salt small and also very nearly equal to half the maximum absorption. The procedure is to take readings with the container in position (full of the salt to be measured) first alone, then with a small radium tube of known strength placed first just in front and then just in back of the container. The ratio of the first reading to the mean of the other two gives the ratio of the strength of the unknown to the sum of the strengths of the known and unknown within one per cent. The method is both quicker and more accurate than the emanation method for concentrations between 10^{-5} and 10^{-7}. The container used was made of tin plate and was 10 cm. by 10 cm. by 2 cm. thick, the radii being 30 and 32 cm.

INTRODUCTION.

THE determination of the radium content of highly concentrated radium preparations by means of the gamma-ray method is so well developed that an accuracy of less than one per cent. can be reached in any laboratory where a standardized radium preparation is available. The degree of accuracy becomes less and less when products of lower concentration are to be measured: this is due to the absorption of the gamma rays in the salt itself and also to the fact that the dimensions of these preparations usually are not so small that the differences in distance of the various points of the preparation from the center of the ionization vessel can be neglected. The ordinary gamma-ray methods give sufficient accuracy for salts of 100 per cent. to 1 per cent. radium content (concentration 1 to 10^{-2} g. Ra/g.). If the absorption coefficient of the gamma rays within the salt is known approximately and the absolute amount of salt taken does not exceed about 10 g., sufficient accuracy may be also obtained with salts of 10^{-3} or 10^{-4} g. of radium per gram of salt. For these cases it suffices to assume that the absorption of gamma

rays of radium distributed uniformly throughout the substance is equal to the absorption in a layer of half of the actual thickness of the preparation and so make a correction accordingly. But it is not possible to apply the gamma-ray method to salts of very much lower concentration than 10^{-4} g./g. using the customary apparatus: with quantities of 10 g. of salt or less the absolute amount of Radium present is too little to be measured accurately and taking much more salt increases the influence of the absorption within the salt too much. For salts with less than 10^{-4} g. Ra/g. therefore it has been necessary so far to use the emanation method. This method is much less accurate than the gamma-ray method and involves the necessity of dissolving or fusing the salt. With salts of medium concentration (10^{-5} to 10^{-7} g. Ra/g.) it is furthermore necessary to dilute and this brings in a new source of error, not to speak of inconvenience and difficulties of the emanation method in general (difficulties in obtaining saturation current, inconstancy of radium standard solutions, etc.).

Thus it is quite obvious that any extension of the range of the gamma-ray method would be advantageous. The influence of the absorption of gamma rays in the salt itself was computed by E. Schweidler[1] and H. Thirring[2] for the case where the salts are filled into spherical containers of moderate sizes. The correctness of the results depends upon how accurately in any individual case the coefficient of absorption is known. This naturally varies with the size of the crystals and with the method of filling the container. It certainly would not increase the convenience and accuracy of the gamma-ray method when in any case the coefficient of absorption had to be determined by separate experiment.

We found it possible to extend the range of the gamma-ray method for low-grade salts by taking shallow layers of salt within containers of relatively large areas placed in such a way that all points of the surface of the container are at the same distance from the gamma-ray instrument.

PRINCIPLE OF THE METHOD.

This was accomplished by the following arrangement (Fig. 1). The container was a curved tin plate box 10 cm. high and 2 cm. deep. The radius of curvature was 30 cm. and according to this the container was placed at a distance of 30 cm. from the center of the ionization chamber of the electrometer. This was of cylindrical shape so that the surface of the container formed a part of a concentric cylinder plane. The instrument is a string electrometer (electric capacity 1 cm.) devised by one of the authors in 1913 for gamma-ray measurements.

[1] E. Schweidler, Physik. Zeitschr., Vol. 13, p. 453.

[2] H. Thirring, Physik. Zeitschr., Vol. 13, p. 266, 1912.

It can be foreseen that with such an arrangement the gamma-radiation of a homogeneous radium-barium salt filling the total space between the two cylindric surfaces of the container cannot be very different from the average of the gamma-ray action obtained with the same absolute

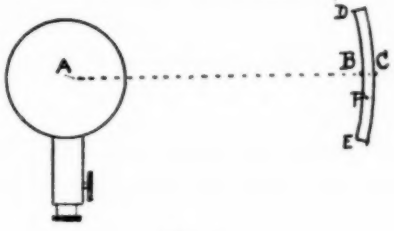

Fig. 1.

amount of radium in form of a little standard tube fixed on the outside of the container, front and back (points B and C, Fig. 1). It is easy to prove this by calculation:

We take P as an optional point between the two cylindric planes of the container and call $AP = r$, $AB = a$, $AC = a + d$; d represents the thickness of the active layer within the container. We consider the action of the gamma rays of the salt between BC upon the center A of the ionization vessel. The front surface of the container (DBE) is called s. Since d is small compared with the distance AB the volume of the container is $s \cdot d$ and the total amount of radium within the container

$$m = \rho \cdot s \cdot d,$$

where ρ denotes the amount of radium per unit of volume. We call furthermore the coefficient of absorption of the gamma rays of RaC within the radium-barium salt μ and the coefficient of absorption in the material of the wall (zinc) μ'.

Taking a shallow elementary layer (enclosed between two concentric cylinder planes at the distance r and $r + dr$) we obtain the ionization caused by this layer as

$$dI = \frac{K \cdot \rho \cdot s \cdot dr \cdot e^{-\mu(r-a)-\mu' d'}}{r^2},$$

where K denotes the number of pairs of ions caused per ccm. and per sec. by 1 g. of radium (gamma radiation only) at 1 cm. distance (the so-called "Eves number"). $e^{-\mu' \cdot d'}$ denotes the absorption in the zinc plate which forms the front plane of the container (actual thickness $d' = 0.35$ mm.). Since r varies but between a and $a + d$ and we have the square of r in the denominator we may take the mean value $(a + d/2)$ instead of r

as an approximation. The total radiation of the container is then given by the integral

$$I = \frac{K \cdot m \cdot e^{\mu a - \mu' d'}}{(a + d/2)^2 \cdot d} \cdot \int_a^{a+d} e^{-\mu r} \cdot dr$$

and the solution is

$$I = \frac{K \cdot m \cdot e^{-\mu' \cdot d'} \cdot (1 - e^{-\mu \cdot d})}{\mu \cdot (a + d/2)^2 \cdot d}.$$

The same amount of radium m concentrated in form of a small tube fixed at B (Fig. 1, front of container) would give a gamma-ray action of

$$\frac{K \cdot m}{a^2}$$

and the same tube fixed at C (rearside of container)

$$\frac{K \cdot m}{(a + d)^2} \cdot e^{-\mu d - 2\mu' d'},$$

because in this case the gamma rays have to traverse a thickness d of salt and twice the thickness of the zinc plate.

The average of these two ionizations is

$$I' = \frac{m \cdot K}{2} \cdot \left[\frac{1}{a^2} + \frac{e^{-\mu d - 2\mu' d'}}{(a + d)^2} \right].$$

In order to compare I and I' it is not necessary to know the specific ionization K caused by the gamma rays of one gram of radium in our apparatus. Taking the same quantity of radium in both cases and forming the quotient I/I' we would expect that this quantity is equal to unity if the principle of our method holds.

Substituting numerical values according to the conditions of our experimental arrangement: $a = 30$ cm., $d = 2$ cm., $d' = 0.035$ cm., $\mu = 0.05$ cm.$^{-1}$, $\mu' = 0.35$ cm.$^{-1}$ we obtain

$$\frac{I}{I'} = 0.9914.$$

In other words: considering the action of radium distributed homogeneously in our curved container upon the center of the ionization vessel we would expect 1 per cent. less ionization as the average of ionization corresponding to two readings with the same quantity of radium filled into a small tube fixed first to the front and then to the rear plane of the container.

Experiments.

The container was filled with an inactive barium salt and a very small radium tube, containing 0.87 mg. radium element imbedded in the salt at different positions throughout and the ionization measured. The average of these readings was 0.2025 scale divisions/sec. Then the radium tube was taken out and fixed at the center of the front and rear side of the container: there we obtained the readings[1] 0.2232 and 0.1858 ScD/sec. The mean of these two readings, 0.2045, agrees within one per cent. with the average of all readings within the salt, given above. The agreement with the calculation is perfect.

Procedure of Measurements with this Method and Results as Compared with Other Methods.

The procedure of actual determinations of radium content of samples by our method will be best illustrated by an example:

The container was filled with 350 grams of a low-grade radium barium carbonate the radium content of which was unknown, and then hermetically sealed. The first reading was taken not before 3 hours after sealing in order to be sure that the short-lived products RaA–RaC are in equilibrium with the emanation.

All measurements were taken at the same distance (30 cm.) from the center of the gamma-ray instrument.

First simply the gamma-ray action of this filled container was measured and we call this ionization i_1. Then the standard tube (0.87 mg.) was placed in addition to that at the center of the front and afterwards at the center of the rear side of our container. We call the ionization observed in these two cases i' and i'' and take their average

$$i_2 = \tfrac{1}{2} \cdot (i' + i'').$$

The current i_1 corresponds to the action of the unknown quantity of radium x within the container, the current i_2 to x plus the standard. Thus we obtain x, expressed in milligrams from the equation

$$\frac{x + 0.87}{x} = \frac{i_2}{i_1}.$$

Dividing the result x by the known total weight of the radium-barium salt in the container we obtain the radium content of the sample in mg. Ra per kilogram of salt. If the salt is not in radioactive equilibrium the same procedure is repeated several days later and from the known law of recovery of emanation in radium salts the end value computed in the usual way.

[1] In fact these figures are average figures of at least 5 readings.

In our special example we obtained by this method the result: Radium content of our sample

1.10 mg. per kilogram of salt.

Now this result was checked by two other methods: In the first place a weighed sample of the salt was dissolved in hydrochloric acid, diluted to a proper concentration and then tested by the emanation method. We obtained a result of

1.05 mg. radium per kilogram of salt.

Another weighed sample of our carbonate was dissolved in concentrated sulfuric acid and again tested by the emanation method. With this sample we obtained

1.11 mg. Ra per kilogram of salt.

The agreement of our method with these other methods is as good as can be expected from the possible limits of error of the emanation methods.

Our method has some advantage over the emanation methods: it takes much less time (just two gamma-ray determinations on two different days are necessary) and no decomposition or dilution of the sample is required.

Our method may be used for all materials the Radium content of which lies between 10^{-5} and 10^{-7} g. Ra per gram of material. It fills the gap between the ordinary range of the gamma-ray method and of the emanation method. For materials with less than 10^{-7} g. Ra/g. the emanation method is convenient because it is then no more necessary to dilute the dissolved sample. A remarkable feature of our method is that knowledge of the coefficient of absorption in the salt used is not required.

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THE VARIATION OF THE PHOTOELECTRIC CURRENT WITH THICKNESS OF METAL.

BY OTTO STUHLMAN, JR.

SYNOPSIS.

Silver and platinum deposited in the form of transparent and semitransparent wedges were examined for variation of the photoelectric current as thickness of the metal was increased. The metal was deposited on quartz by the evaporation method and examined when monochromatic light fell on the metal side of the plate.

The results are found to be consistent with the view that the probability of an electron going a given distance without losing its ability to escape falls off exponentially with the distance, when this distance is less than about 40 $\mu\mu$. Up to this thickness there is a parallelism between optical absorption and photoelectric emission. For large wave-lengths this parallelism is more pronounced than for short wave-lengths. For greater wave-lengths optical absorption may increase, but photoelectric emission decreases terminating at the threshold value of photoelectric sensitivity.

The results seem to support the view that photoelectric emission is probably not caused by the absorption of energy from the incident light beam, the light only acting as the agent which sets the electron free from its parent atom.

TO test any theory of the photoelectric effect it is necessary to know how the photoelectric current varies with the thickness of the metal through which the electrons must pass and the depth to which the light energy penetrates in the metal. To obtain an adequate conception of the laws governing the penetration of both light and electrons it was thought of primary importance to investigate the change in photoelectric current as the metallic film was increased in thickness.

It was hoped, furthermore, that the results of such an investigation might afford a check on the simple assumption that the number of photoelectrons which escape falls off exponentially with the distance moved in reaching the surface of the metal.

The investigation of the behavior of photoelectrons in passing through metals is confined to a few articles[1–7] which in general agree amongst themselves only in the order of the magnitude of the constants involved. The literature[8] presenting the optical point of view is, however, extensive

[1] "Density Law of Absorption," Lenard, Ann. d. Phys., 12, p. 730, 1903.
[2] Ladenburg, Ann. d. Phys., 12, p. 558, 1903.
[3] Crother, Phil. Mag., 12, p. 379, 1906.
[4] Rubens u. Ladenburg, Ber. d. D. Phys. Ges., 24, p. 749, 1907.
[5] Partzsh u. Hallwachs, Ann. d. Phys., 41, p. 247, 1913.
[6] Stuhlman, PHYS. REV., 13, p. 132, 1919.
[7] K. T. Compton and Ross, PHYS. REV., 13, p. 374, 1919.
[8] For a summary see Partzsh and Hallwachs, Ann. d. Phys., 41, p. 250, 1913.

though often very contradictory. Whether the light penetrates the metal to a depth of several wave-lengths or only a fraction of a wave-length is still an open question.

Method.—The results were obtained with the apparatus outlined in the writer's above-cited paper[6] and are a continuation of this work and an effort to extend and correlate similar problems investigated by K. T. Compton, Ross, Robinson, Partzsh and Hallwachs and by the writer.

The metals, platinum and silver, were deposited by the evaporation method[1] in the form of wedges, transparent at their thin ends and opaque, or nearly so, at their thick ends. These semitransparent wedges deposited in quartz were then examined for their photoelectric effect at successive millimeter intervals along their lengths, when light from a 110-volt Cooper-Hewitt quartz mercury vapor lamp was allowed to fall at normal incidence on the metal side. A quartz lens of about 20 cm. focal length was used with a slit, which appeared as an image one millimeter in width, when focussed on the metallic wedge under examination. When monochromatic light was used, a Hilger quartz monochromatic illuminator was interposed.

Results.—The curve *ABC* in Fig. 1 shows the characteristic photoelectric emission obtained from a metal as a function of the thickness of the metal, when exposed to the full radiation from the quartz mercury vapor lamp, at normal incidence. Somewhat similar results, though only qualitatively interpretable, were previously published by J. Robinson,[2] and the writer working with K. T. Compton.[3] One possible theoretical treatment of the problem was worked out by Partzsh and Hallwachs.[5] They suggested that if I_p is the intensity of the light which enters and penetrates the surface of the metal, then at a depth x this intensity drops to $I = I_p e^{-\alpha x}$, where α is the coefficient of absorption of the light. From any thickness dx located some distance x below the surface of the metal $nIdx$ electrons start. These are absorbed exponentially in passing back through the thickness x with coefficient of absorption β. The total number emerging is

$$N = nI_p \frac{1 - e^{-d(\alpha+\beta)}}{\alpha + \beta},$$

where d is the thickness of the metal under examination. Three tacit assumptions are involved, namely that the number of electrons which retain their ability to escape falls off exponentially with the distance moved *normally* to the surface, and that the coefficients of absorption of

[1] Stuhlman, Jour. Optical Soc., 1, p. 78, 1917; PHYS. REV., 13, p. 112, 1919.
[2] Robinson, Phil. Mag., 25, p. 115, 1913; ibid., 32, p. 421, 1916.
[3] Stuhlman and Compton, PHYS. REV. (2), II., p. 208, 1913.

the light and of the electrons are independent of the thickness of the metal.

This simple theory leads one to expect the photoelectric current to increase exponentially from zero thickness, to a saturation value comparable to the current obtained from a thick sheet of metal under the usual circumstances. The thickness at which this occurs is then the maximum depth to which the light penetrates the metal.

The extensive curve shown in Fig. 1 was not obtained from a single metallic wedge. At first a wedge very rapidly increasing in thickness was examined. This information gave the general outline of the curve. Then a very thin wedge was examined. This gave the data for the points lying at the beginning of the curve. These detailed data usually covered about the first third of the graph as shown in Fig. 1. Then this wedge was replaced in the deposition apparatus and more metal was deposited, thus increasing the thickness uniformly over the whole surface. Upon examination this second wedge usually furnished the data for the second third of the graph. This was repeated until the saturation value of the photoelectric current curve was obtained. A check on the shape and location of the sink in the curve was obtained by placing two quartz plates end to end and depositing a metal wedge on both simultaneously. In this way the deposit on one plate gave the data for one part of the curve and the deposit on the other the data for the adjoining part.

The thickness of the metal at some point along the wedge was obtained as follows: A quartz plate was covered with a uniform deposit of the metal and matched in color with a similar point on the wedge, to identify its relative position. The photoelectric current from this plate, when exposed under the same conditions as the wedge, was compared with the photoelectric current from the wedge. The thickness of the uniformly deposited plate was determined by weighing, and this thickness identified the thickness of the metallic wedge which produced the same photoelectric current as the uniformly deposited plate.

Where no points of observation are shown on the graphs, it is understood that a combination of several of the above methods furnished the data. Where extrapolations or interpolations are resorted to, the curves are shown by broken lines.

The dotted line in Fig. 1 shows the results to be expected from the above theory on the assumption that the two coefficients are of the same order of magnitude and independent of the thickness of the metal. The experimental data do, in general, conform to this theory especially for large wave-lengths. For the data presented above, however, there exists a marked departure from the simple theory. While the experi-

mental results usually fit the theory for very small and large thicknesses of metal, for intermediate thicknesses there is a marked departure as at *ABC*. Here the curve is seen to pass through a pronounced minimum, *B*, before reaching its saturation value.

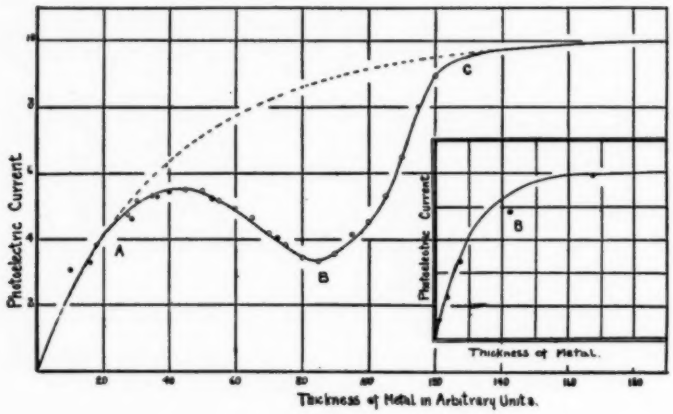

Fig. 1.

The earlier work by the writer and K. T. Compton[11] and J. Robinson[10] also showed an unusual change in the photoelectric current for metallic thicknesses comparable to 10^{-7} cm., apparently out of all proportion to the amount of light absorbed by the metal. Robinson's data stops just short of the point *B* in Fig. 1, hence he interpreted his results as showing an abnormal emission of current for thicknesses comparable to the point *A*. That is, his results are interpreted as showing a maximum in the curve at *A* while the above curve and subsequent data show that the photoelectric current proceeds normally as far as the point *A* and then departs from the theoretically expected current by an apparent subnormal development between *A* and *C*. Beyond *A* the photoelectric current seems to be no longer proportional to the thickness of the metal penetrated by the incident light energy.

In support of this interpretation are quoted some results from an earlier paper by the writer and K. T. Compton[11] shown as an insert in Fig. 1. This curve represents the photoelectric current emitted by a platinum film, cathodically deposited in successively thicker layers on a platinum plate. As in the above curve the current rises exponentially from zero thickness to its maximum, or saturation, value with a small but appreciable variation at the point *B*. This point of observation, at that time attributed to experimental error, lies below the exponential curve drawn through the other points. Here, as before, we meet with a subnormal current, not an abnormal rise in current, for thickness of

metal comparable to 10^{-7} cm. Hence with the completed curve before us we must conclude that the photoelectric current is proportional to the thickness of the metal for all values of thickness except such critical values as give an apparent subnormal current.

Results With Monochromatic Illumination.—The work was next extended to determine the change in the photoelectric current with change in frequency of the incident light. Some characteristic results as obtained from silver are shown in Fig. 2. The upper curve shows the photoelectric current from silver as a function of metallic thickness when unresolved light from the quartz mercury vapor lamp fell on the metallic side of the wedge at normal incidence. It shows the approximate average location of the subnormal part of the curve referred to as *B* in Fig. 1. A cover-glass one millimeter thick was next introduced into the path of the incident beam. Previous examination with a Hilger monochromatic illuminator had shown that this plate absorbed all wave-lengths below λ 3131. The results thus obtained are given by the lower curve of Fig. 2. A Hilger monochromatic illuminator was next introduced into the incident beam and set for λ 2536. These results are shown by the middle curve.

Increasing the wave-length of the incident light-energy shifts the subnormal part of the curve to regions of greater thicknesses.

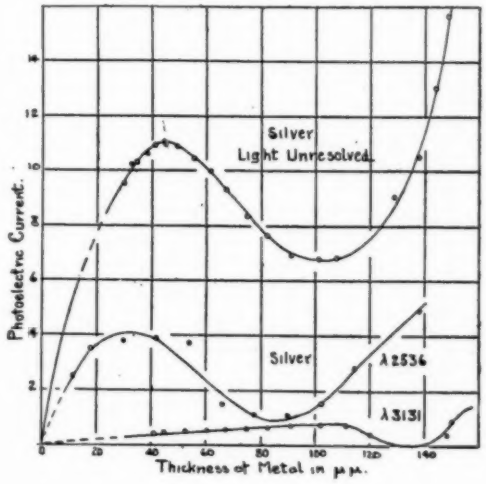

Fig. 2.

A similar comparison with platinum as given in Fig. 3 shows the same effect only to a less degree. If we compare platinum with silver, for wave-length λ 253.6 for wedges deposited under identical conditions, we find that the subnormal position of the curve for silver lies at thickness 90 $\mu\mu$ while platinum has its minimum at about 45 $\mu\mu$ thickness.

Results Due to Fatigue.—It was thought that the slight shift in the minimum of the curve as the wave-length of the incident light-energy decreased was due to aging because of occlusion of gases. Shrinkage in thickness due to aging might account for it, but this possibility was

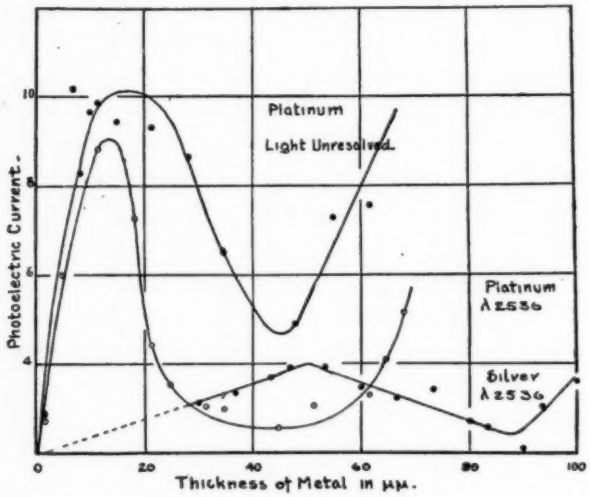

Fig. 3.

eliminated when the thickness of the metal was determined immediately after the photoelectric current measurements. To test for any change caused by aging due to absorption of gases, a silver wedge was examined directly after deposition (curve No. 1, Fig. 4), then twenty-four hours

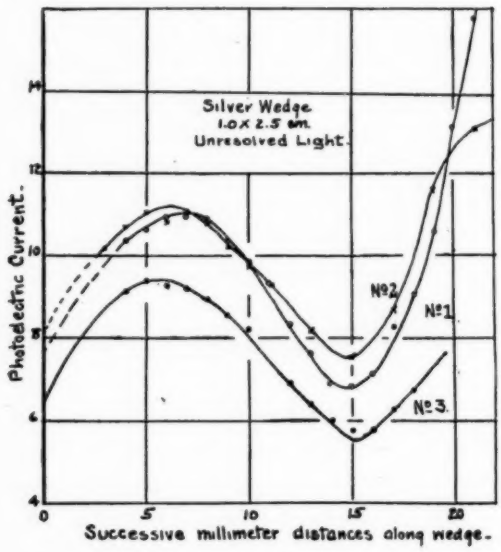

Fig. 4.

later. These latter results are shown by curve No. 2. Then the metal was examined eight days later with the results shown in curve No. 3. These curves show no measurable shift in the location of the minimum, although there is an indication of a slight shift in the position of the peak of the curve, towards the thin end of the wedge. The relative changes in magnitude of the current follow the usual course of changes which accompany "photoelectric fatigue." On the other hand a more detailed examination, of this possible change, with platinum and wave-length 253 $\mu\mu$, with a nine-day idle period between tests, showed a slight shift of the peak in the curve towards the region of greater thickness. No measurable change in the position of the minimum point, except that which could be accounted for by experimental error, was detected.

We might conclude therefore that for any given metal there is a real departure from the simple saturation curve showing that the thickness of the metal and photoelectric effect are proportional and that this departure appears as an apparent subnormal photoelectric current. The subnormal part of the curve or point of minimum photoelectric emission in the curve shifts to regions of greater thickness as the wave-length of the exciting light increases.

Discussion.—All the curves at the origin have one thing in common, they are either linear or possess a slight curvature concave downward. Robinson's results[10] show his curves as possessing a point of inflexion near the origin, as if the photoelectric current increased at first slowly and then more rapidly with thickness of metal. Theoretically the photoelectric emission and thickness are proportional, as there is a proportionality between absorption of light and thickness of metal.[7, 2]

If the amount of light absorbed is proportional to the resulting photoelectric current then this proportionality is only true for the first 10 $\mu\mu$ thickness of the metal penetrated by the light in case of very short waves, but may reach a thickness of 300 $\mu\mu$ or more for wave-lengths comparable to λ 3130. This initial depth in which exponential absorption takes place, as manifested by the emission of photoelectrons, increases as the wave-length of the incident light-energy increases. It cannot however increase as rapidly as the wave-length increases due to the decrease in kinetic energy of emission of the electrons as the wave-length approaches its photoelectric threshold value.

If the coefficient of absorption for silver as determined by Fritze[1] and later by W. Planck[2] are examined, for wave-lengths lying between 400 and 500 $\mu\mu$, we find in general an exponential increase in these

[1] Fritze, Ann. d. Phys., 47, p. 763, 1915.

[2] W. Planck, Phys. Zeit., p. 563, 1914.

coefficients with increase in thickness of the metal through which the light penetrates. Their results show a depression in the coefficient of absorption-thickness curve in the vicinity of 50 $\mu\mu$ thickness for λ 546 and what appears to be a shift in this depression to smaller thicknesses as the wave-length decreases to λ 526.

If we examine the absorbing power of these two metals or their transparency, with increasing thickness, we at once notice several interesting analogies. Silver and platinum as examined by Hagen and Rubens for their relative transmission, for a given wave-length as a function of the thickness of the metal, found that a silver surface 80 $\mu\mu$ thick became more and more transparent as the wave-length of the incident energy decreased from λ 7000 to λ 3210. At the latter wave-length the silver transmitted 32 per cent. of the incident energy. From this wave-length down to λ 2210 the surface rapidly became less transparent, reaching a minimum of 1.5 per cent. for wave-length 2210.

Upon comparing equal thicknesses of metal, say 80 $\mu\mu$, we find as shown in Fig. 2, that with decreasing wave-length, the photoelectric results show an increase in the transmission of light by silver. This transmission reaches a maximum at λ 3210. Here the absorption is a minimum but not quite zero. Photoelectrically the light absorption ceases at λ 3250, the threshold value for silver. Beyond this wave-length photoelectric action does not set in again although optical absorption increases very rapidly. If absorption of light and emission of photoelectrons go hand in hand, why does not the photoelectric effect follow the increase in optical absorption? It seems as if the two phenomena at this point were only remotely related or have nothing in common. It appears probable that there are really two phenomena superimposed; the optical absorption which removes energy from the light-wave, and the photoelectric effect which removes no energy from the light-wave. The light appears to act only as the agent reaching into the atom to release a particular electron.

Hagen and Rubens' optical results showed that silver possesses marked regions of selective transmission; a given thickness becoming more and more transparent as the light decreased in wave-length from λ 3160 to λ 2510. This is also verified by the above photoelectric measurements. Where the photoelectric current passes through a minimum the metal must allow most of the incident energy to pass through it without producing a photoelectric effect, so that the light transmitted by this layer is the same as the amount transmitted by a much thinner layer of metal. Under these circumstances it would be impossible to use the transmitted energy as a measure of film thickness as has been done by Compton and Ross.[7]

If we compare the results from silver with those from platinum, say for wave-length λ 2536, we at once notice that the photoelectric current for platinum is very much larger than that for silver, when equal thicknesses of metal are compared. This is also in agreement with the optical absorption of these metals. Hagen and Rubens[1] found that for λ 3210 silver was about one thousand times more transparent than platinum when equal thicknesses of metal (80 $\mu\mu$) were compared. Judging from the photoelectric results and using the assumption that the photoelectric current is proportional to the amount of light absorbed, this magnitude seems very much too large. At most the photoelectric results show platinum only ten times more absorbing than silver. This variation might be accounted for by the great difference in the absorption of the electrons as they pass through the metals. Unfortunately the coefficient of absorption of the electron for platinum is not more than twice as large as that for silver.[2] Again we are confronted with the same difficulty, in that the optical absorption does not allow us to arrive at a conclusion as to the magnitude of the photoelectric current that may be liberated by a metal.

Finally an examination of the photoelectric current for a change in the intensity of the light showed a corresponding change in the magnitude of the current but no measurable shift in the curve along the thickness axis.

Conclusions.

The amount of light absorbed by a metal and the photoelectric current generated as the result of the penetration of the light into the metal are not in general proportional to each other.

If the light penetrating a metal is absorbed according to the known simple exponential law then the photoelectric current is only proportional to the amount absorbed in a depth of metal comparable to less than one tenth a wave-length of the incident energy.

The greater the wave-length of the energy producing photoelectric activity the greater the depth to which absorption of light and emission of current are proportional.

In the case of silver the variation in optical transmission runs parallel to the variation in the photoelectric current as the thickness of metal changes.

Optical absorption of the light may take place without photoelectric emission.

Photoelectric emission is not caused by the absorption of the light

[1] "Durchlässigkeit" as used by Hagen and Rubens, Ann. d. Phys., 8, p. 449, 1902.

[2] Stuhlman, Phys. Rev., 15, p. 549, 1920.

energy, but the light acts only as an agent in setting the electrons free from their parent atoms.

The number of photoelectrons retaining ability to escape falls off exponentially with the distance moved in reaching the surface of the metal. The data presented could not however be used to distinguish between this conclusion and one in which the ability to escape falls off exponentially with the distance moved normally to the surface.

THE UNIVERSITY OF NORTH CAROLINA,
CHAPEL HILL.

NOTE ON THE RATIO OF MASS TO WEIGHT FOR BISMUTH AND ALUMINUM.

By H. A. Wilson.

Synopsis.

The ratio of mass to weight has been shown to be the same for bismuth and aluminum to within one in one million, by the method of Eötvös.

MR. BRUSH[1] has recently described some experiments on the gravitational attractions exerted by several substances and on the ratio of mass to weight for some of them. Contrary to the results of previous experimenters he finds this ratio different for different substances and the attractions per unit mass also different. Mr. Brush's experiments appear to have been conducted with some care and it is not easy to suggest sources of error which might account for his surprising results. In the case of his pendulum experiments he does not state that he corrected for the buoyancy of the air and this effect, if not allowed for, would be sufficient to produce an error of the same order as the differences he found.

Mr. Brush found the greatest difference in the attractions in the case of aluminum and bismuth, so I have made a comparison of the ratios of mass to weight for these two substances by the very excellent method of Eötvös.[2] The result shows that this ratio is the same for these two substances to within one part in one million.

Two small cylinders, one of bismuth and one of aluminum, and each weighing about one gram were attached to the ends of a horizontal aluminum wire six cms. long which was soldered to a vertical wire at its middle point. The vertical wire was suspended by a fine platinum wire, thus forming a torsion balance. A mirror was carried by the vertical wire and the balance was suspended in a suitable metal case having a glass window. The case could be turned about a vertical axis on a stand provided with levelling screws. A telescope, provided with illuminated cross wires in its eye-piece was attached to the case and the apparatus could be adjusted so that the image of the cross wires formed by light reflected from the mirror could be made to coincide with another cross wire in the eye-piece.

[1] Phys. Rev. (2), XIX., p. 125, 1921.

[2] Mathematische und Naturwissenschaftliche Berichte aus Ungarn, Bd. 8, S. 64, 1891.

The case was turned so that the balance beam was approximately East and West and was then turned through two rightangles. It was found that after turning through two rightangles the image of the cross wires came to rest not more than one quarter of a millimeter from its previous position in the eye-piece. The lower end of the vertical wire of the balance dipped into a small vessel containing castor oil which gave sufficient damping. To prevent twisting of the torsion wire when the case was turned through two rightangles stops were provided which carried the beam round with the case. These stops were put in position after the telescope was adjusted so that the image coincided with the cross wires and were removed after the case had been rotated.

The time of one complete vibration of the beam was about three minutes.

The theory of the experiment is as follows. Owing to the rotation of the earth part of the attraction of the earth on a body is used up in keeping the body on its circular path.

If r denotes the radius of the earth and ω its angular velocity, then in latitude θ, the horizontal component of the centrifugal force on a body of mass m is $mr\omega^2 \cos\theta \sin\theta$. The attraction of the earth is mg towards its center so that if ϕ denotes the inclination of the apparent weight to the earth's radius

$$\phi = \frac{r\omega^2 \cos\theta \sin\theta}{g}.$$

For another body suppose

$$\phi' = \frac{r\omega^2 \cos\theta \sin\theta}{g'}.$$

If the two bodies have nearly equal masses and are suspended East and West on a torsion balance, as in Eötvös' experiment, there will be a couple acting on the balance equal to $mgd\ (\phi - \phi')$ where d is half the length of the balance.

If T is the time of one oscillation of the balance $T = 2\pi d\sqrt{2m/\mu}$, where μ is the torsion coefficient of the wire. Hence $\mu = 8\pi^2 md^2/T^2$. The couple $mgd\ (\phi' - \phi)$ will twist the wire through an angle Δ given by

$$\mu\Delta = mg\ (\phi' - \phi)\ d.$$

When the apparatus is turned through two rightangles the wire will be twisted through an angle 2Δ and

$$2\Delta = \frac{2mg\ (\phi - \phi')\ d}{\mu} = \frac{(g - g')\ T^2 r\omega^2 \cos\theta \sin\theta}{4\pi^2 g'}.$$

If t denotes the time of one revolution of the earth then $\omega = 2\pi/t$ and

$$\frac{g - g'}{g'} = (2\Delta)\ \frac{2d\pi^2}{rT^2 \sin 2\theta}.$$

For Houston $\theta = 30°$ so with $T = 180$ secs. $g = 980$ and $d = 3$ cms. we get

$$\frac{g - g'}{g'} = 25 \times 10^{-4} \times (2\Delta).$$

The experiments showed that 2Δ was less than about 1/2500, so that $(g - g')/g'$ was less than about 10^{-6}.

We conclude therefore that the acceleration of gravity is the same for bismuth and aluminum to within one part in one million.

Rice Institute,
Houston, Texas,
March 25, 1922.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE WASHINGTON MEETING, APRIL 21 AND 22, 1922.

THE 115th regular meeting of the American Physical Society was held at the Bureau of Standards in Washington, D. C., on Friday and Saturday, April 21 and 22, 1922. There were morning and afternoon sessions on both days, with an attendance of about one hundred and seventy-five members and visitors. The presiding officers were Theodore Lyman and Charles E. Mendenhall, President and Vice-President of the Society, respectively. The Association of Apparatus Manufacturers held sessions upon the same days. On the evening of Friday, April 21, there was a joint dinner, held in the dining room of the Bureau of Standards, at which there were addresses by Director S. W. Stratton, Professor R. A. Millikan and Professor F. K. Richtmyer. There was also a very interesting exhibit of scientific apparatus.

There was a meeting of the Council on the afternoon of Friday, April 21, 1922, attended by fourteen members. There were important discussions regarding the report of the Educational Committee and with regard to the development of the PHYSICAL REVIEW. One candidate was elected to Fellowship and thirty-two were elected to membership, as follows: *Elected to Fellowship*; Otto Maass: *Elected to Membership*; Norman I. Adams, Donald S. Ainslie, George W. Alderman, O. S. Duffendack, David C. Duncan, Scott B. Ewing, O. Rex Ford, George S. Gessner, George E. Gibson, Helen G. Rafsky, A. L. Greenlees, William W. Harper, Kalman Heindlhofer, August Hund, G. E. M. Jauncey, Robert V. Kleinschmidt, Raymond G. Lehnhoff, James B. Macelwane, W. C. McQuarrie, Duncan MacRae, George R. Miller, Leslie F. Murch, Brian O'Brien, W. H. Pielemeier, Samuel Robinson, Allen G. Shenstone, James J. Smith, Robert B. Sosman, M. N. States, Alma T. Waldie, Usaburo Yoshida, J. F. T. Young, W. C. Geer, Walter S. Huxford, Edwin L. Rose.

The following program of sixty-two papers was presented; seven of these, Nos. 28, 29, 33, 42, 43, 56 and 60 were read by title:

1. Characteristic X-Ray Absorption in the "L" Series for Elements N62 to N77. J. M. CORK.

2. Effect of Temperature on the X-Rays Scattered by Crystals. G. E. M. JAUNCEY.

3. The Effect of Occluded Hydrogen upon the Crystalline Space-Lattice of Palladium. L. W. McKeehan.

4. X-Ray Limits Beyond the Range of Spectroscopic Measurements. F. L. Mohler and Paul D. Foote.

5. Precision Measurements of the Alkali Halides. Wheeler P. Davey.

6. Total Reflection of X-Rays from Glass and Silver. Arthur H. Compton.

7. The Crystal Structure of Strontium Selenide. Mabel K. Slattery. Introduced by Edna Carter.

8. A New Method of Using X-Rays in Crystal Analysis. William Duane and George L. Clark.

9. A Study of Secondary Valence by Means of X-Rays. George L. Clark and William Duane.

10. The Scattering of X-Rays at Small Angles. A. R. Duane and William Duane.

11. Note on the Size of the Electron as Determined by the Absorption and Scattering of X-Rays. F. K. Richtmyer.

12. Short Electric Waves. E. F. Nichols and J. D. Tear.

13. The Optical Constants of Certain Liquids for Short Electric Waves. J. D. Tear.

14. A Mechanical Model for Hysteresis. A. Press.

15. The Variation of the Photoelectric Current with Thickness of Metal. Otto Stuhlman, Jr.

16. Free Modes of Oscillation in Loop Aerials. S. L. Brown and C. P. Boner.

17. Improved Experiments on Magnetization by Rotation. S. J. Barnett and L. J. H. Barnett.

18. The Effect of a Longitudinal Magnetic Field on the Mechanical Properties of a Vibrating Nickel Wire. C. V. Kent and B. J. Babbitt. Introduced by H. M. Randall.

19. Theory of End-Loss Corrections and Their Application to Tungsten Filaments in Vacuo. A. G. Worthing.

20. Effect of the Current Distortion by Long Lines on the Operation of Recording Relays. Carl Kinsley.

21. The Calculation of the Capacity of Antennas. Frederick W. Grover.

22. Direct Capacity Measurement. George A. Campbell.

23. The Complete Scale of Color Temperature and its Application to the Color Grading of Daylight and Artificial Illuminants. Irwin G. Priest.

24. A Simple and Accurate Method for Determining Surface Tension and Density of Molten Glass. Edward W. Washburn.

25. New Proof of the Specular Reflection of Molecules. R. A. Millikan and Karl S. Van Dyke.

26. Radial Flow in Rotating Liquids. Harold Mottsmith and Irving Langmuir.

27. Noises in Stressed Metals. Barry MacNutt and A. Concilio.

28. The Simple Rigidity of Drawn Tungsten Wire at Incandescent Temperatures. WILLIAM SCHRIEVER.

29. The Reversible Inductivity of Rochelle Salt under High Frequency Fields. JOHN G. FRAYNE.

30. Reciprocal Diffraction Relations Between Circular and Elliptical Plates. JOHN COULSON and G. G. BECKNELL.

31. A Flow Calorimeter for Measuring the Heat Capacity of Gases. H. F. STIMSON, T. S. SLIGH and N. S. OSBORNE.

32. Elastic Constants of Solid Rods at High Frequencies. W. G. CADY.

33. The Influence of the Surface Tension on the Efflux of a Liquid in Jet-Form. JULIUS HARTMANN. Introduced by Hugo Fricke.

34. A Study of the Luminous Discharge in Hydrogen and in Mercury and a New Method of Measuring Ionization Potentials. GEORGE E. GIBSON and W. ALBERT NOYES, JR.

35. Excitation of the Spectra of Hydrogen and of Nitrogen in Low-Voltage Arcs. O. S. DUFFENDACK. Introduced by K. T. Compton.

36. The Relative Intensities of Stark Effect Components in the Helium Spectrum. JOHN STUART FOSTER.

37. Cathodoluminescence of Heated Alumina. D. T. WILBER.

38. Radiation from Oxides. I. Emissive Power in Red of Zirconium Oxide. W. E. FORSYTHE.

39. Variation of Contact Difference of Potential with Temperature in Potassium and Sodium, and the Accompanying Changes in Photo-Electric Emission. HERBERT E. IVES.

40. A Precision Rotating Sector: Aperture Variable and Measurable While in Motion. A. H. PFUND.

41. Specific Intensity of Radiation in an Absorbing Medium. LEIGH PAGE.

42. Phenomena in Gases Excited by Radio Frequency Currents. E. O. HULBURT.

43. On the Effect of Short Electromagnetic Waves on a Beam of Electrons. C. J. LAPP.

44. Effect of the Initial Emission Velocities of Electrons on the Minimum Arcing Voltage in Gases. K. T. COMPTON and Y. T. YAO.

45. The Independence of the Arcing Voltage in Mercury Vapor on the Nearness of the Liquid Surface. Y. T. YAO.

46. Mobilities of Electrons in Hydrogen. LEONARD B. LOEB.

47. The Electron Emission from Thoriated Tungsten Filaments. IRVING LANGMUIR.

48. The Removal of Thorium from the Surface of a Thoriated Tungsten Filament by Bombardment with Positive Ions. K. H. KINGDON and IRVING LANGMUIR.

49. The Measurement of Magnetic Field Strength by Means of Electron Tubes. ALBERT W. HULL.

50. A General Relation for Electron Emission from Metals. SAUL DUSHMAN.

51. The Secondary Electron Emission from Nickel. C. DAVISSON and C. H. KUNSMAN.

52. An Experimental Method of Testing the Expressions for Longitudinal and Transverse Masses of the Electron. L. T. JONES and W. C. POMEROY.

53. Oil Drop Experiments as Proof of the Invariance of Electric Charge. F. W. LOOMIS.

54. Study of the Hydrogen Radiation in the Red and the Infra-Red. F. S. BRACKETT.

55. The Voltage-Current Relation in Central Anode Photo-Electric Cells. HERBERT E. IVES and THORNTON C. FRY.

56. The Broadening of the Balmer Lines of Hydrogen with Pressure. E. O. HULBURT.

57. Crystal Structures of Vanadium, Germanium and Graphite. ALBERT W. HULL.

58. The Diffusion Coefficient in Solids and Its Temperature Coefficient. SAUL DUSHMAN and IRVING LANGMUIR.

59. Note on the Cohn-Minkowski Formula for the Motion of an Insulator in a Magnetic Field. S. J. BARNETT.

60. A New Method for the Generation of Sound Waves. JULIUS HARTMANN. Introduced by Hugo Fricke.

61. The Effect of a Magnetic Field on the Absorption of X-Rays. JOSEPH A. BECKER. Introduced by F. K. RICHTMYER.

62. Formation and Life of Metastable Helium. FABIAN M. KANNENSTINE.

The abstracts of papers in the above program, with corresponding numbers, are given on the following pages.

DAYTON C. MILLER,
Secretary.

1. CHARACTERISTIC X-RAY ABSORPTION IN THE "L" SERIES FOR ELEMENTS N62 TO N77.

BY J. M. CORK.

USING photographic method and the Tungsten emission lines as standard, the wave-length of the three L bands was determined.

The square root of frequency plotted with atomic[1] numbers (combined with measurements of other observers), give lines with curvature upward for increasing atomic number.

Combination of the absorption frequencies with that of the emission lines[2] help to determine the energy levels in such modifications of the Bohr-Sommerfeld atom as proposed by Wentzel and Smekal.

With such energy levels established a uniform notation for emission lines which would tell their origin, is suggested.

UNIVERSITY OF MICHIGAN.

[1] Hertz, Z. für P., 3, p. 19. Duane, P. N. A. S., 6, p 509. DeBroglie, C. R., 170, p. 585.
[2] Hjalmar, Z. für P., 7, p. 341.

2. Effect of Temperature on the X-Rays Scattered by Crystals.

By G. E. M. Jauncey.

According to Debye (Ann. der Phys., Band 43, 1914, pp. 47–95) the intensity of the x-rays scattered by a crystal at a given angle is proportional to $1 - e^{-M}$, where M is a function of the ratio of the temperature of the crystal to the characteristic temperature.

Applying Debye's formulæ to calcite, for which A. H. Compton gives the characteristic temperature as 910° K., and assuming 20 as the average atomic weight of the atoms of the calcite molecule, the ratio of the intensity of the scattered radiation at 70° with the primary rays when the temperature of the crystal was 565° K. to that when the temperature was 290° K. was calculated on the assumption of (1) no zero-point energy (2) zero-point energy. Assuming a wave-length of .30 Å.U. the values of the ratio are (1) 2.22 and (2) 1.32.

A calcite crystal was heated to the above temperatures and the ratio observed was 1.05 with certainly an error no more than 15 per cent. The effective wave-length of the rays was about .30 Å.U. It would appear that Debye's theory is incorrect. This experimental result is supported to some extent by a theoretical paper by L. Brilloun (Ann. de Phys., Jan.–Feb., 1922, p. 88) in which he throws some doubt on the theory of Debye.

Washington University,
St. Louis, Mo.,
April 6, 1922.

3. The Effect of Occluded Hydrogen upon the Crystalline Space-Lattice of Palladium.

By L. W. McKeehan.

The crystalline space-lattice of palladium is face-centered cubic, and the edge of the unit cube is about 3.90×10^{-8} cm. Hydrogen is readily absorbed by thoroughly annealed metal at room temperature and atmospheric pressure, and increases the parameters of the space-lattice by as much as several per cent., without altering its type. There is evidence, however, that the space-lattice is locally distorted by the introduction of the hydrogen atoms. Whether these take the places of palladium atoms or occupy intermediate positions cannot as yet be determined because of the great differences in weight and scattering power between atoms of the two elements.

Research Laboratories,
Western Electric Company.

4. X-Ray Limits beyond the Range of Spectroscopic Measurements.

By F. L. Mohler and Paul D. Foote.

Differences in absorption limits adequately explain all x-ray emission lines of heavy elements. The authors have extended computations of limits to light elements thus supplementing data obtained from critical potential

measurements below the range of x-ray spectroscopy. It has been shown before that the K series starts with the ionization potential of helium and the limits L_1 and L_2 from neon. For light elements we found other L limits softer than L_1 and these too give by combination K emission lines. Kurth's measurements of radiation from solids instead of gases indicated single L limits for aluminum and silicon much higher than L_1. These two points fall on the extrapolated L_3 line. We suggest the hypothesis that radiation from outer L orbits is suppressed in solids just as arc radiation is. For this reason the emission spectra observed by Millikan may belong to the L_3 series. Extrapolation of the M series shows that the ionization point of argon and the softest x-ray limits of potassium and nickel as measured by the authors, and of iron and copper (as measured by Kurth) belong to the series M_1M_2. Three higher potentials measured by Kurth fall close to M_5.

BUREAU OF STANDARDS,
April 5, 1922.

5. PRECISION MEASUREMENTS OF THE ALKALI HALIDES.

BY WHEELER P. DAVEY.

USING the method described at the February meeting for making precision measurements of the x-ray diffraction patterns of crystals, the dimensions of the halides of Cs, Rb, K and Na have been determined with an accuracy of 0.1 per cent. The data presented will be published shortly in full in the PHYSICAL REVIEW. The data show two interesting features. (1) The difference in "distance of nearest approach" of centers of $\overset{+}{\text{Cs}}$ and $\overset{+}{\text{Rb}}$ in the iodide, bromide, and chloride is constant to within experimental error. The same statement may be made of $\overset{+}{\text{Cs}}$ and $\overset{+}{\text{K}}$, but not of $\overset{+}{\text{Cs}}$ and $\overset{+}{\text{Na}}$. The difference in "distance of nearest approach" of centers of $\bar{\text{I}}$ and $\bar{\text{Br}}$ in the Cs, Rb and K salts of these halogens is constant to within experimental error. The same statement may be made of $\bar{\text{I}}$ and $\bar{\text{Cl}}$. (2) Since, in the case of the Cs compounds, the "distance of nearest approach" is measured along the cube-diagonal of the crystal, and in the case of the Rb and K compounds it is measured along the cube-edge of the crystal, it appears from the constant values obtained for (CsI–CsBr), (RbI–RbBr) and (KI–KBr) that $\bar{\text{I}}$ and $\bar{\text{Br}}$ act as though they were spherical in shape. A similar statement can be made of $\bar{\text{Cl}}$.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

6. TOTAL REFLECTION OF X-RAYS FROM GLASS AND SILVER.

BY ARTHUR P. COMPTON.

IF the frequency of the radiation traversing a medium is considerably greater than the natural frequency of vibration of any of the component electrons, the index of refraction of the medium should be $n = 1 - Ne^2/2\pi mf^2$. Here N is the number of electrons per unit volume, e and m have their usual values, and f is the frequency of the radiation. Since this index is less than unity, such radiation should be totally reflected from a polished surface if the angle of incidence is greater than the critical angle.

Experiments have been performed which show that regular reflection of x-rays does occur at angles of a few minutes of arc from polished surfaces of glass and silver, and that except near the critical angle the greater part of the incident energy is reflected. The critical angles for the reflection of $\lambda = 1.279$ Å.U. from plate glass and silver mirrors have been measured, giving values of $1 - n = 5.0 \times 10^{-6}$ and 20.9×10^{-6} respectively. Taking the number of electrons per atom to be the atomic number, the corresponding theoretical values are 5.2 and 19.8×10^{-6}. The agreement is thus quite satisfactory.

WASHINGTON UNIVERSITY,
SAINT LOUIS,
April 5, 1922.

7. THE CRYSTAL STRUCTURE OF STRONTIUM SELENIDE.

BY MABEL K. SLATTERY. INTRODUCED BY EDNA CARTER.

SrSe was prepared by heating the selenate to a red heat in a stream of hydrogen. Its x-ray diffraction pattern was photographed according to the powder method. The thirteen lines obtained fit a simple cubic lattice with $d = 3.10$ Å. Density measurements show the unit structure to be a cube of dimensions 3.10 Å., the alternate corners being occupied by Sr and Se ions. This structure is similar to that of KCl and RbBr.

VASSAR COLLEGE,
POUGHKEEPSIE, N. Y.

8. A NEW METHOD OF USING X-RAYS IN CRYSTAL ANALYSIS.

BY WILLIAM DUANE AND GEORGE L. CLARK.

IN this method either a small crystal (a volume of .25 cubic mm. suffices) or crystal powder may be employed.

In analyzing a small crystal we place it on the axis of rotation of an x-ray spectrometer, fix the ionization chamber at some convenient angle, 2θ, from the zero line of the instrument and send through it a beam of rays belonging to the continuous x-ray spectrum (not the line spectra). We then turn the crystal through a series of small angles and measure the ionization current at each step. When any one of the crystal's principal planes makes the angle θ

with the incident beam it picks out the x-rays of wave-length λ, given by the equation

$$n\lambda = 2d \sin\theta \qquad (1)$$

and reflects them into the ionization chamber, producing a marked increase in the current. The curves shown represent the ionization current as a function of the angular position of the crystal in an experiment with a potassium iodide crystal. The angular distances between the sharply marked peaks on the curves are the angles between the respective reflecting planes in the crystal.

To get the distance between successive planes in any one set we place the crystal so that this set of planes reflects the x-rays into the ionization chamber and then reduce the voltage applied to the tube. When the voltage reaches the value, V, given by the equation

$$Ve = h\nu = \frac{hc}{\lambda},$$

it no longer can produce x-rays of length λ, and, for the first order spectrum, $n = 1$, practically vanishes. The point at which the current vanishes appears to be sharply marked. The values of V and θ substituted in equations (1) and (2) give the required distance, d. The only other quantity that enters into the computations is the universal constant $hc/e = 12{,}354$, if V is in volts and λ in Ångströms.

Our experiments with the potassium iodide crystal indicate a simple cubic structure (in agreement with Davey and Wyckoff), the length of the cube's edge being 3.532×10^{-8} cm. Assuming one atom of each element to a cube this gives for the density 3.111, in close agreement with the value measured by Baxter, 3.114.

If the crystal is in the form of powder, we place it at the center of the spectrometer, fix the ionization chamber at some angle, 2θ, as above, and measure the ionization current with different voltages applied to the tube. The curve representing the ionization current as a function of the voltage has breaks in it at voltages corresponding to the vanishing of the x-rays reflected by the different sets of crystal planes. Equations (1) and (2) give the distance between successive planes in each set.

HARVARD UNIVERSITY.

9. A STUDY OF SECONDARY VALENCE BY MEANS OF X-RAYS.

BY GEORGE L. CLARK AND WILLIAM DUANE.

THE ability of electrically neutral molecules to combine with other molecules capable of existing free, such as inorganic salts with water to form hydrates, with ammonia to form ammines and with molecular halogens to form polyhalides, is attributed to secondary valence forces residing in the stray fields of atoms and ions. The uncertainty and confusion in this subject has led to the present researches. Numerous remarkable relationships between the stabili-

ties of secondary valence complexes and the dimensions of the atoms and molecules comprising them have been established and will be published in a separate paper. The only possible method of correlation of these relationships and of discovering how the complexes are formed and constituted is analysis by means of x-rays. The first compound studied in this way was potassium tri-iodide, $KI.I_2$. The crystal preserved against sensible decomposition by very special precautions was analyzed by the x-ray method described in the previous paper. It was found to be centered cubic, with an iodine *atom* going to the center of each of the original unit cubes of KI and thereby expanding them from an edge length of 3.532 to 4.680×10^{-8} cm. Experimentally $n = \rho d^3/\text{Mol. Wt.} \times \text{Wt. H atom}$, where n is the number of molecules per unit cube, and ρ the density, was found to be .4986 or $\frac{1}{2}$. At an angle of 6° 45′ the ionization current vanished at a voltage of 11,400. The structure accounts for a great many of the properties of $KI.I_2$. Studies are now in progress on $CsI.I_2$ where CsI itself is centered cubic with a value of d of 4.562×10^{-8} cm., upon other polyhalides and upon some ammines and hydrates beginning with those of the lithium halides.

HARVARD UNIVERSITY.

10. THE SCATTERING OF X-RAYS AT SMALL ANGLES.

BY A. R. DUANE AND WILLIAM DUANE.

MANY of the experiments performed in this laboratory during the last eight years on the short wave-length limit of the continuous x-ray spectrum show that the ionization current in the ionization chamber of an x-ray spectrometer becomes exceedingly small, when the angle made by the chamber with the direct beam of rays has been reduced to a value less than that at which the crystal can reflect the rays. The experiments with crystal powders described in the papers presented at this meeting by G. L. Clark and one of us show the same effect. All of these results indicate that the amount of radiation scattered by the reflecting substance at angles less than that of reflection becomes very minute in comparison with the reflected radiation.

The object of the research described in this paper has been to test various substances for this phenomenon. We fix the material to be examined at the center of an x-ray spectrometer, and measure the ionization current with the chamber in different positions. The curves in the slide shown give the currents as functions of the angles in an experiment with a sheet of aluminium. The currents represent the secondary and scattered radiation from the aluminium. The curves indicate that this radiation reaches a well-defined maximum at an angle of about 13° from the direction of the primary beam of rays, and that, as this angle decreases, the radiation also decreases, reaching a very small value at a sharply marked angle of 9°.7. Making corrections for the natural leak of the instrument and for stray radiation we find that the radiation from the aluminium at angles smaller than 9°.7 can not amount to as much as one per cent. of that at the maximum.

On the assumption that the radiation at angles greater than 9°.7 is largely due to reflection from the planes containing the aluminium atoms we can calculate the maximum distance between these planes. The equation

$$Ve = \frac{hc}{\lambda}$$

gives us the minimum wave-length, λ, in the x-ray beam, and the equation

$$\lambda = 2d \sin \theta$$

gives us the distance, d, between the planes. A correction for the widths of the slits, amounting to 0°.8, has to be added to 9°.7 in order to get the true angle. Substituting half of the angle 9°.7 + 0°.8 and the value of the voltage, V, used in the experiment, namely 29,350 volts, in the equations, we get $d = 2.32$ Ångströms. The greatest distance between the planes in aluminium given by A. W. Hull is 2.33, in close agreement with our value.

It appears from this research that the scattering from such a substance as aluminium may be treated as largely due to reflection from planes. Many other substances have been examined with similar results,[1] for instance water and wood.

HARVARD UNIVERSITY.

11. NOTE ON THE SIZE OF THE ELECTRON AS DETERMINED BY THE ABSORPTION AND SCATTERING OF X-RAYS.

BY F. K. RICHTMYER.

J. J. THOMPSON's expression for the absorption and scattering of x-rays

$$\frac{\mu}{\rho} = KN^4\lambda^3 + \frac{\sigma}{\rho}$$

has been verified by the writer and others down to $\lambda = .09$ Å. Extrapolation for aluminum to $\lambda = 0$ gives a value of $\sigma/\rho = .15$, some 25 per cent. smaller than the theoretical value $(8\pi/3)(e^4/m^2)n = .20$ as derived by Thompson. Whereas the value of μ/ρ for gamma rays is known to be of the order of .05. Accordingly A. H. Compton modifies the above expression to

$$\frac{\mu}{\rho} = \phi_f N^4\lambda^3 + \phi_s \frac{\sigma_0}{\rho}$$

where ϕ_f and ϕ_s are coefficients whose value depends on the ratio of the wave-length of the x-rays to the diameter of the (ring) electron, which Compton computes to be 1.85×10^{-10} cm.

[1] During the progress of our research a paper by Hewlett appeared in the March number of the PHYS. REV., p. 267, in which he mentions having observed a reduction of the radiation scattered from carbon at small angles. Further, an interesting report on scattering of x-rays has been sent to us by Professor A. H. Compton, in which he suggests an explanation of Hewlett's observation similar to that we proposed for our results. Professor Compton's report will be published shortly by the National Research Council.

Data on aluminum, previously reported by the writer, and more recently by Duane, give values of the mass absorption coefficient which are 7.5, 25, 50, and 90 per cent. higher than Compton's computed values at wave-lengths, respectively, of .20, .15, .125, and .10 Å., although Compton's formula and the writer's data agree for $\lambda > .30$ Å. This indicates an electron much smaller than computed by Compton.

Taking the scattering coefficient, as determined by extrapolation of data over the region $.20 < \lambda < .40$ Å., at .15, and using Compton's formula, an electron of diameter 1×10^{-11} cm. gives a scattering coefficient of .057 at $\lambda = .005$ Å.

CORNELL UNIVERSITY.

12. SHORT ELECTRIC WAVES.

BY E. F. NICHOLS AND J. D. TEAR.

METHODS for the generation and isolation of radiation of a few millimeters' wave-length have been developed. The generator finally adopted is based upon the principle of the Hertzen oscillator. It embodies a number of new features making possible the generation of very short waves.

The receiver is the Nichols radiometer. Receiving elements of either 1 μ platinum wire of proper length for resonance with the incident radiation, or of platinum films deposited by distillation upon thin mica are substituted for the usual blackened radiometer vanes. By proper choice and adjustment of vanes, the receiver may be made either selective or non-selective.

For securing greater homogeneity, a reflecting echelon made of carefully machined brass blocks spaced at intervals of $\lambda/2$ was used and found to reflect selectively. Two such reflecting echelons with steps at right angles when placed one after the other in the path of the beam give, under suitable conditions, a fair approximation to monochromatic radiation. Mountings for these echelons have been designed easy of adjustment to any desired spacing. Even one such echelon can be made to give an approximate analysis of the incident radiation.

At present the shortest wave-length isolated is 1.8 mm. The shortest observed, but not reproducible at will, is 0.8 mm.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
March 29, 1922.

13. THE OPTICAL CONSTANTS OF CERTAIN LIQUIDS FOR SHORT ELECTRIC WAVES.

BY J. D. TEAR.

THE coefficients of reflection and absorption of water, glycerin, methyl and ethyl alcohol for the wave-lengths 4, 8, and 18 mm. have been measured, and the refractive indices computed by the Cauchy-Quincke formula.

A second measure of the refractive index has been obtained in a few cases by measuring the reflection from a thin layer of the liquid over a mercury surface.

As the thickness of the layer is varied, the reflected energy passes through maxima and minima giving a measure of the wave-length in the liquid.

The results as obtained by the two methods agree within the limits of experimental error.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
March 29, 1922.

14. A MECHANICAL MODEL FOR HYSTERESIS.

BY A. PRESS.

THE model comprises a slider spring constrained with respect to a frame. The slider is frictionally mounted, adjustably upon the frame runway. Mounted on the slider is a movable yoke, spring constrained with respect to the slider. The motion of the yoke with respect to the frame registers the magnetic displacement B with respect to the force H impressed on the yoke. The stored energy in the slider spring corresponds to the energy of I and the energy in the yoke spring that due to H (for a vacuum). The frictional constraint of the slider corresponds to the hysteresis loss factor and is proportional to I and not H. Various types of loops are obtained by making the frictional coefficient a function of the slider displacement by means of wedge-shaped surfaces. Magnetic saturation is simulated by affecting the displacement factor for the slider springs. The model furnishes a complete analog of mechanical and electrical as well as magnetic hysteresis.

WASHINGTON, D. C.

15. THE VARIATION OF THE PHOTOELECTRIC CURRENT WITH THICKNESS OF METAL.

BY OTTO STUHLMAN, JR.

SILVER and platinum deposited in the form of transparent and semitransparent wedges were examined for variation of the photoelectric current as thickness of the metal was increased. The metal was deposited on quartz by the evaporation method and examined when monochromatic light fell on the metal side of the plate.

The results are found to be consistent with the view that the probability of an electron going a given distance without losing its ability to escape falls off exponentially with the distance, when this distance is less than about 40 $\mu\mu$. Up to this thickness there is a parallelism between optical absorption and photoelectric emission. For large wave-lengths this parallelism is more pronounced than for short wave-lengths. For greater wave-lengths optical absorption may increase but photoelectric emission decreases terminating at the threshold value of photoelectric sensitivity.

The results seem to support the view that photoelectric emission is probably not caused by the direct absorption of energy out of the incident light beam,

the light only acting as the agent which sets the electron free from its parent atom.

THE UNIVERSITY OF NORTH CAROLINA.

16. FREE MODES OF OSCILLATION IN LOOP AËRIALS.

BY S. L. BROWN AND C. P. BONER.

LOOPS with the turns widely separated have been investigated to ascertain their free modes of oscillation when excited by a spark in the loop circuit, by damped oscillations in a circuit loosely coupled with the loop, and by undamped oscillations. The results show that a loop has several free modes of oscillation when excited by a spark in the loop circuit and that the higher modes have no simple relation to the lowest or fundamental mode. The intensity of these modes of oscillation may be altered by adding capacity across the terminals of the loop but this capacity affects the frequency of the lowest or fundamental mode only, to any appreciable extent. When a loop is excited by damped oscillations in a nearby circuit, it will respond to several different frequencies of the exciting current, but these different resonant frequencies are changed by adding capacity across the terminals of the loop. When the loop is excited by undamped oscillations in a nearby circuit, it will respond to several different frequencies of the exciting current, but the intensity and frequency of the several resonant oscillations are changed by adding capacity across the terminals of the loop.

UNIVERSITY OF TEXAS.

17. IMPROVED EXPERIMENTS ON MAGNETIZATION BY ROTATION.

BY S. J. BARNETT AND L. J. H. BARNETT.

SINCE the report of Dec., 1920, more than two thousand observations have been made on iron, steel, nickel, cobalt, and Heusler alloy. To reduce errors from speed, spiral gears have been substituted for belt and pulleys. The magnetometer has been improved, and has been used altogether in four different positions. Small stellite journals, an agate bearing, and an improved process of demagnetization, together with a return to equatorial positions of the magnetometer, have been used to reduce the error from torsion. To reduce errors from eddy currents, a larger and better constructed frame has been used to compensate the main part of the earth's magnetic field; and extra coils, in conjunction with variometers, have been installed to compensate the variations. Other errors have been reduced or eliminated. A long-suspected systematic error has been found in the 1917 magnetometer observations, causing the results to differ considerably from those obtained by the method of electromagnetic induction in 1914 and 1915, now fully confirmed. Rotating any one of the ferromagnetic substances named at one revolution per second is equivalent to placing it in an axial magnetic field with intensity -3.5×10^{-7} gauss, within 10% or less. This result would be expected if

a single type of negative magneton, with ratio of angular momentum to magnetic moment equal to m/e, one half that of a ring of electrons, were responsible for ferromagnetism. Abraham showed in 1903 that a superficially charged Lorentz electron in rotation has the moment and the momentum giving this ratio. If a ring of electrons is chiefly responsible for ferromagnetism, positive magnetons also participate. Important improvements are in progress.

CARNEGIE INSTITUTION OF WASHINGTON.

18. THE EFFECT OF A LONGITUDINAL MAGNETIC FIELD ON THE MECHANICAL PROPERTIES OF A VIBRATING NICKEL WIRE.

BY C. V. KENT AND B. J. BABBITT. INTRODUCED BY H. M. RANDALL.

A NICKEL wire is set in free longitudinal vibration in a magnetic field. The change of elastic modulus with field intensity (H) at various tensions is calculated from change in frequency as determined by the method of Lissajou's figures. The retraction is also measured. The results agree well in character and magnitude with those of Honda and Terada.

The decrement of vibration varies in a remarkable manner for different fields and tensions.

For a given tension, the changes in modulus and decrement, and the rate of change of retraction, reach maximum values at the same field intensity. For different tensions, these maxima occur at nearly the same value of magnetization intensity, but different field intensities.

UNIVERSITY OF MICHIGAN.

19. THEORY OF END-LOSS CORRECTIONS AND THEIR APPLICATION TO TUNGSTEN FILAMENTS IN VACUO.

BY A. G. WORTHING.

A SOLUTION has been obtained for the differential equation giving the temperature distribution in an electrically heated, uniform, cylindrical filament in vacuo near a cooling lead or support, which is applicable to both long and short filaments.

For long filaments those in which the cooling effect at any point depends on a single lead or support, it is found that, when temperatures are expressed in terms of the maximum central temperature and distances in terms of what might be called a reduced unit of length, a single curve suffices for any temperature distribution. The same is approximately true for resistivity, luminosity, thermionic emission, etc. Total corrections are similarly related.

For short filaments, those in which the cooling effect at any point depends on two leads or supports, it is found that similar relations hold for filaments with equivalent lengths.

For filaments having different cross sections, *e.g.*, ribbon or wire, it is found that equivalent filaments—those operating at the same temperature and wattage on the same voltage—suffer equal end-losses.

All three conclusions have been verified experimentally to within the uncertainties of measurement.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
April, 1922.

20. EFFECT OF THE CURRENT DISTORTION BY LONG LINES ON THE OPERATION OF RECORDING RELAYS.

BY CARL KINSLEY.

THE arrival curve of a current impulse is distorted by a long line. It is seen from the Braun tube oscillograph shown that a recording device with any lost motion will give very inaccurate results. Extensive experiments by J. Zelisco have shown that with the principal relays in service a time interval of 0.002 to 0.005 second is needed to set the relay tongue into motion and from 0.004 to 0.006 second is then used before the record is made.

A relay which can be fitted to the circuit in which it is to be employed, which in use has satisfied all the requirements yet imposed and which makes an autographic record of its operations, has been recently developed and can be briefly described.

A schematic drawing shows the essential details. The tongue is composed of a stiff spring with a soft iron armature on its end. The natural period can be made as high as desired and should be considera bly above that of the succession of impulses whose record is sought. The tongue is permanently magnetized—by a magnet or local current—and is normally held in a distorted position. The line impulse releases the tongue, with no time loss, and the record is made at an interval after the release, which can be exactly determined by computation as is shown by the graph giving frequency tests. The local circuit is closed through a moving chemicall y prepared paper strip which records the instant of contact, the time of which can be determined by a clock record made on the same strip. In the illustration the records of two such relays are given when operating at a speed of 0.005 second between strokes. No variation in operation within 0.0001 second is found on the record.

CORNELL UNIVERSITY,
March 30, 1922.

21. THE CALCULATION OF THE CAPACITY OF ANTENNAS.

BY FREDERICK W. GROVER.

FORMULAS have long been available for the calculation of the capacity of wires, where the length is very great compared with the radius of cross section and the distance between the axes. These conditions are not satisfied in the case of systems of wires such as are used for antennas. It is necessary in such cases to take into account the variation of the charge density along the wire resulting from finite length.

Formulas applicable to such systems have been published in recent years by Howe and Cohen. The former obtains the capacity by calculating the potential of the charge under certain assumptions as to the effect of the variation of the charge density; the latter makes use of the reciprocal relation between the linear inductance and capacity of the wires and formulas for the joint inductance of parallel circuits. Howe has pointed out the considerable differences between the values given by the two methods.

The present paper shows that the disagreement of the methods is only apparent and disappears if accurate inductance formulas are used. Simplified formulas are also obtained for calculating the capacity of a number of important arrangements of wires, including multiple systems, and tables are given which enable numerical results to be obtained with a minimum of labor.

UNION COLLEGE,
SCHENECTADY, N. Y.

22. DIRECT CAPACITY MEASUREMENT.

BY GEORGE A. CAMPBELL.

THE direct capacities of an electrical system with given accessible terminals are defined as the capacities which, connected across the terminals in pairs, will be the exact equivalent of the system in its external reaction upon any other electrical system with which it is associated only by conductive connections through the accessible terminals. Direct capacities are thus Maxwell's "coefficients of induction," with sign reversed, but they have the advantage of invoking the picture of a concrete network of condensers which is a realizable equivalent of the given electrical system. Direct conductances, admittances and impedances are defined in a similar manner. An individual direct capacity, the magnitude of which is required, cannot be removed and isolated from the other direct capacities in its own electrical system, but must be measured while still connected to them. Methods of measurements have, however, been devised which give a direct reading of the individual direct capacity. Of these methods, the one found to be most accurate, convenient and generally useful is a substitution method involving a particular connection of the terminals of the electrical system to the corners of an alternating current bridge containing variable capacity and conductance or their equivalent.

AMERICAN TELEPHONE AND TELEGRAPH COMPANY.

23. THE COMPLETE SCALE OF COLOR TEMPERATURE AND ITS APPLICATION TO THE COLOR GRADING OF DAYLIGHT AND ARTIFICIAL ILLUMINANTS.

BY IRWIN G. PRIEST.

THE scale of color temperature has been proposed by Hyde and his associates as a suitable scale for the color grading of illuminants, and considerable data on the color temperatures of artificial illuminants have been published.[1] The

[1] Hyde and Forsythe, J. Frank. Inst., *183*, pp. 353–354; 1917. Hyde, Forsythe and Cady,

experimental methods used by Hyde and his associates for the determination of color temperatures have been restricted to temperatures below about 3200° K., *i.e.*, below the temperature of the arc and far below the various phases of daylight.

The purposes of the present work have been to devise means for the color grading of daylight and artificial daylight as well as artificial illuminants in terms of color temperature and to establish a color-temperature scale from the color of the Hefner lamp to that of the blue sky (1880° to 24,000° K.).

Using the method of rotatory dispersion,[1] it has been found possible to determine the apparent color temperature of the various phases of daylight. It has been noticed, however, that the spectral center of gravity of light is a more suitable scale than temperature, since on it equal intervals indicate approximately the same color difference at all points on the scale, which is very far from true on a temperature scale.

The method is described, its sensibility, precision and accuracy are discussed, and experimental data on the apparent color temperature of daylight together with a diagram of the scale are given in a paper now in course of preparation.[2]

NATIONAL BUREAU OF STANDARDS,
April 1, 1922.

24. A SIMPLE AND ACCURATE METHOD FOR DETERMINING SURFACE TENSION AND DENSITY OF MOLTEN GLASS.

BY EDWARD W. WASHBURN.

Surface Tension.—The glass is contained in an electrically heated porcelain pot about 9 cm. in diameter and 15 cm. high provided with an electrically heated cover, in the center of which is a hole through which passes a platinum wire, on the lower end of which, just above the surface of the glass, hangs a platinum cylinder 4 cm. in diameter by 4 cm. high. The weight of the cylinder is borne by a sensitive coiled spring. The level of the lower end of the spring is read with a cathetometer and the furnace and pot are then gradually raised until the surface of the glass makes contact with the platinum cylinder and a second reading is taken with the cathetometer. The platinum cylinder is then pushed below the surface a short distance and allowed to rise to its equilibrium position. By this procedure the equilibrium position is found to be the same for the two directions of approach as closely as can be determined with a cathetometer. The platinum cylinder is standardized before and after each determination by similar measurements with liquids of known surface tension.

Density.—The density of molten glass is determined with the same apparatus,

PHYS. REV. (2), *10*, pp. 395–411; 1917. Forsythe, PHYS. REV. (2), *18*, p. 147; 1921. Hyde and Forsythe, Trans. I. E. S., *16*, pp. 419–427; 1921. Kingsbury, J. Frank. Inst., *183*, pp. 781–782; 1917.

[1] Cf. Priest, J. Op. Soc. Am., *5*, pp. 178–183; 1921. B. S. Sci. Papers, No. 417; 1921. J. Op. Soc. Am., Jan., 1922.

[2] "The Color Grading of Daylight and Artificial Daylight in Terms of Color Temperature by the Method of Rotatory Dispersion," *Sci. Pap., Bureau of Standards.*

but with a platinum sphere substituted in place of the platinum cylinder. The largest source of error in the measurements is that arising from the variability of the temperature.

Results.—The results show that the surface tension of the soda-lime-silica glasses between 1200° and 1450° C. is of the order of magnitude of 150 dyne/cm.

UNIVERSITY OF ILLINOIS.

25. NEW PROOF OF THE SPECULAR REFLECTION OF MOLECULES.

BY R. A. MILLIKAN AND KARL S. VAN DYKE.

THE results obtained by one of the authors in previous studies by the oil-drop method have now been fully confirmed by measurements made on the coefficient of slip by the constant-deflection method using gases at low pressures enclosed between concentric cylinders, one of which is rotated at constant speed. The fraction of the impinging molecules which are specularly reflected and the fraction diffusely reflected are determined for different gases and different surfaces and the results found in complete agreement with the results previously found by the oil-drop method.

CALIFORNIA INSTITUTE OF TECHNOLOGY.

26. RADIAL FLOW IN ROTATING LIQUIDS.

BY HAROLD MOTTSMITH AND IRVING LANGMUIR.

ZOCHER has observed that when a colloidal solution of vanadium pentoxide in a beaker is given a rotary motion and examined vertically between crossed nicols, a dark cross appears whose arms, however, are not parallel and perpendicular to the plane of polarization, but are displaced 10° to 20° backward against the direction of rotation. It is now shown that this hitherto unexplained displacement is due to a radial component of flow arising from unbalanced centrifugal forces. The liquid near the bottom moves slowly because of viscosity. The more rapid rotation of the free surface causes an outward flow at the top and corresponding inward flow at the bottom. Because of the greater shearing stress at the bottom the double refraction due to the inward radial flow predominates and causes the observed shifting of the cross. A striking demonstration of the radial flow is also furnished by the settling of a precipitate from a liquid in a beaker. If the *solution* is rotated in a stationary beaker the precipitate settles in the center, but if the *beaker* containing a quiescent liquid is suddenly set into steady rotation the precipitate settles near the outer walls.

RESEARCH LABORATORY, GENERAL ELECTRIC CO., SCHENECTADY, N. Y.

27. NOISES IN STRESSED METALS.

BY BARRY MACNUTT AND A. CONCILIO.

A SENSITIVE microphone was attached to test specimens of Sb, Bi, Cd, Sn, Zn, and steel. The specimens were stressed in a testing machine and the

loads at which noises were heard noted. The load at which the first noise was heard was quite constant for a given material and occurred in the neighborhood of the yield point.

LEHIGH UNIVERSITY.

28. THE SIMPLE RIGIDITY OF A DRAWN TUNGSTEN WIRE AT INCANDESCENT TEMPERATURES.

BY WILLIAM SCHRIEVER.

THE simple rigidity of a 10-mil drawn tungsten wire was determined at temperatures between 1000° and 2000° K. A static method was employed. By applying the same torque to both a short length and a long length of wire —middle portions of both lengths at the same temperature—it was possible to get the angle of twist of a section of the wire, all of which was at a uniform temperature. The torque necessary to produce the twist in the heated wire was measured by the twist in a fine cold tungsten wire of known constants. The temperatures were measured by means of an optical pyrometer of the Morse type; corrections were made for the absorption of the evacuated glass tube which enclosed the wire.

The modulus of simple rigidity for an equiaxed wire was found to be 21.7×10^{11} dynes/cm.^{2} at 1000° K. and only 3.1×10^{11} dynes/cm.^{2} at 2000° K. There was a relatively small decrease in the rigidity modulus between room-temperature and 1100° K. At a temperature (1600° K.) where the rigidity of tungsten was equal to that of steel at room temperature, the elastic limit of the tungsten was, relatively, extremely small.

Jeffries'[1] work on the change of crystal structure of tungsten wire by heat-treatments at various temperatures makes it possible to draw the following conclusions from the temperature-rigidity curves of the 10-mil tungsten wire:

1. The rigidity of an equiaxed tungsten wire is greater than that of a freshly drawn wire, both measurements being made at the same temperature.

2. Heat treatment in the grain-growth region causes an increase of rigidity at temperatures between 300 and 2000° K., and probably at all temperatures. Increase in grain-size causes an increase in the rigidity modulus.

Moduli of rigidity, as given in tables of physical constants, have, in general, been obtained from angles of twist which were not great enough to cause the elastic limit of the material to be passed. In the writer's research, the angles of twist were less than 0.10 degree per centimeter length of hot wire and yet, at the higher temperatures, this was considerably beyond the elastic limit. The readings of the angles of twist were made so quickly that the results calculated from them are very similar to those which would have been obtained if a torsion pendulum of a very small period had been used. It is suggested that the moduli of rigidity calculated from such observations be called moduli of "Instantaneous Rigidity."

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA,
July, 1921.

[1] "The Metallography of Tungsten," Trans. Am. Inst. Mining Eng., pp. 1037–1092, 1918.

29. The Reversible Inductivity of Rochelle Salt under High Frequency Fields.

By John G. Frayne.

Reversible inductivity K is defined as $\Delta E \doteq o(-\Delta D/\Delta E)$, D being the induction and E the electric field in the dielectric. A plate of the crystal cut with its faces perpendicular to the axis of greatest piezo-electric activity was used as the dielectric of a condenser. The capacity was determined by a resonance method using a field of two million cycles per second as the exciting source. The value of K is about one tenth the dielectric constant as measured by D.C. methods. The variation of K with temperature was obtained between $-80°$ C. and $+50°$ C. It increases from $-80°$ C. up to a maximum of $-20°$ C., then decreases to a minimum about 4° C., and increases again to a maximum at 23° C., and then decreases until the melting temperature is reached. The concave region of the curve coincides with the range of greatest piezo-electric activity. The dielectric constant as measured at D.C. varies in the opposite manner between $-20°$ and $+23°$ C.

The quantity K was measured for different states of polarization of the crystal. The high frequency e.m.f. was superimposed on the D.C. field. Different samples of the crystal used showed permanent polarization effects. The crystal became conducting and the capacity infinite for smaller field strength in one direction than in the other.

University of Minnesota.

30. Reciprocal Diffraction Relations between Circular and Elliptical Plates.

By John Coulson and G. G. Becknell.

Since the time of Poisson and Arago it has been a well-known phenomenon in light that, in the case of a point source, a bright circular spot exists at the center of the geometrical shadow of a disc placed perpendicular to the wave normal. But attention seemingly has not been called to the succession of diffraction patterns appearing in the shadow when the disc is rotated about an axis in its own plane, nor to the reciprocal figures produced by ellipses.

A series of photographs made by a disc inclined at angles varying from 0° to 60° have been prepared. These illustrate the gradual change from the circular spot into an upright cross and thence into a series of broadening diamond-like figures with concave sides.

At any particular inclination of the circle an apparently identical figure may be produced by a thin ellipse, whose minor axis equals the diameter of the disc, cut from a circular cylinder at the same angle of inclination and placed perpendicular to the wave normal. An ellipse similarly cut, but with its major axis equal to the diameter of the disc, produces a contracted figure of the same shape. When an ellipse is rotated till its shadow is a circle the Arago spot is produced at its center.

The typically quadrantal character of the diffraction figure has drawn attention to the fact that each quarter of it is due to light from a particular quadrant of the circular disc or ellipse. This is analyzed in a number of photographs shown in the paper in which light is excluded from certain portions of the edges of these discs.

These figures have been found to be the evolutes of the corresponding geometrical shadows.

UNIVERSITY OF PITTSBURGH,
PITTSBURGH, PA.

31. A FLOW CALORIMETER FOR MEASURING THE HEAT CAPACITY OF GASES.

BY H. F. STIMSON, T. S. SLIGH, AND N. S. OSBORNE.

THIS instrument, though intended primarily for ammonia, is designed to measure the heat capacity of gases at pressures up to 75 atmospheres and temperatures below 150° C. The design is based on a well-known principle which has been used by numerous experimenters. The method is to observe the temperature rise produced by measured electric power in a steady stream of gas traversing the instrument at a measured rate.

Certain refinements have been attempted with the object of limiting size and heat capacity, securing thermal sensitivity, minimizing thermal leakage, and controlling experimental conditions which affect steadiness. Accessory apparatus includes stirred thermoregulated liquid baths, automatic pressure regulator, orifice flow-controller, pressure manometers, vacuum pumps, etc.

The calorimeter has been used for measurements on ammonia at temperatures from $-$ 15° C. to $+$ 150° C. and at pressures from 0.5 atmosphere to 20 atmospheres. Measurements have been made with rates of gas flow ranging from 0.3 gm./min. to 3 gms./min., power input from 0.1 watt to 1 watt, and temperature rises from 4° C. to 50° C. Measurements can usually be begun within an hour after starting the gas flow. Over 75 measurements of C_p of superheated ammonia vapor in the above range with various flows and temperature rises indicate an order of precision of 0.1 per cent.

BUREAU OF STANDARDS,
April 4, 1922.

32. ELASTIC CONSTANTS OF RODS AT HIGH FREQUENCIES.

BY W. G. CADY.

IN previous papers a method of setting a metallic rod into high-frequency longitudinal vibration has been described, the driving force being derived from small plates of a piezo-electric crystal cemented to the sides of the rod at its center. A high-frequency alternating electric field is impressed upon the crystal plates by means of an oscillating triode, causing alternating mechanical stresses which in turn are transmitted to the rod. In the present investigation this method is applied to the determination of Young's modulus for various

metals and for glass, at frequencies from 15,000 to 100,000, and for quartz, in a direction perpendicular to the optic and electric axes, at frequencies from 100,000 to 2,000,000.

By applying the theory of longitudinal vibrations in rods[1] to data obtained with the above-mentioned resonators by means of a sensitive thermo-element, it has been possible to obtain provisional values for the coefficients of viscosity of different substances at high frequency, and to study the dependence of coefficient of viscosity upon frequency.

WESLEYAN UNIVERSITY,
MIDDLETOWN, CONN.,
March 5, 1922.

33. THE INFLUENCE OF THE SURFACE TENSION ON THE EFFLUX OF A LIQUID IN JET-FORM.

BY JULIUS HARTMANN, DR.TECH. INTRODUCED BY HUGO FRICKE.

BECAUSE of the influence of the surface tension, the velocity of a liquid jet is modified from that found by Torricelli's law, and is given by the expression

$$V^2 = \left[\frac{\pi}{4} k^2 d^2\right]\left[h - \frac{1}{Pg} \cdot \frac{2C}{d}\right] 2g,$$

where V = the velocity = the volume of the outflowing liquid per second,
d = the diameter of the jet,
k = the co-efficient of contraction of the jet,
h = head,
p = specific gravity of liquid,
c = capillary constant.

An experimental arrangement for the verification of this formula has been made and the above formula is found to hold good for mercury and for water. The method furnishes a means of accurately measuring the capillary constant. The values found have been:

Water C = 74.0 dynes/cm. at 10° C.
Mercury C = 536.0 dynes/cm. at 16° C.

THE ROYAL TECHNICAL COLLEGE,
COPENHAGEN.

34. A STUDY OF THE LUMINOUS DISCHARGE IN HYDROGEN AND IN MERCURY AND A NEW METHOD OF MEASURING IONIZATION POTENTIALS.

BY GEORGE E. GIBSON AND W. ALBERT NOYES, JR.

THE luminous discharge in mercury and in hydrogen at various pressures was studied in tubes of various dimensions using heated tungsten cathodes. If the discharge is started in such a tube and the potential between anode

[1] PHYS. REV., *19*, p. 1, 1922.

and cathode is then diminished, the discharge disappears suddenly at a voltage which is reproducible, provided the temperature of the cathode is sufficiently high. The value of this disappearance e.m.f. was found in hydrogen to be nearly a linear function of the pressure at high pressures. When the pressure is diminished below a certain value, depending upon the dimensions of the tube, the linear relationship between E and p ceases to hold, E diminishing less rapidly with the pressure and coming to a minimum, $E_{\text{min.}}$, at a value of p depending upon the dimensions of the tube.

If the length between anode and cathode is gradually increased in a tube of given diameter, $E_{\text{min.}}$ remains nearly constant at the ionization voltage, (30.8 for hydrogen and 10.6 for mercury) until at a limiting value of the length $E_{\text{min.}}$ rises abruptly to twice the ionization voltage, remains constant at this value over a further range of length, then rises abruptly to three times the ionization voltage and so on.

UNIVERSITY OF CALIFORNIA.

35. EXCITATION OF THE SPECTRA OF HYDROGEN AND OF NITROGEN IN LOW VOLTAGE ARCS.

BY O. S. DUFFENDACK.

Hydrogen.—In a two-element tube the lines and bands appeared with the arc, but the bands did not appear when the gas was dissociated in the tungsten furnace. This is additional evidence that the lines are due to the atom and the bands to the molecule.

Nitrogen.—The first positive bands appeared with the arc and increased in intensity with the voltage. The second positive bands appeared with the arc and decreased in intensity with increased voltage. At potentials above 70 volts the arc turned into a brilliant "flare" with enormously increased conductivity. The second positive bands appeared along with the spectrum of tungsten. This was explained on the basis of the formation of "active" nitrogen. The negative bands appeared a volt or two above the ionizing potential and increased markedly in intensity. Three bands not previously reported were observed at 5075, 5018, and 4961 Å. Only four lines were observed, 5003, 5006, 5667, and 5680, and these never below 70 volts. The probability that these are enhanced lines was discussed.

It was concluded that the positive bands are due to the neutral molecule, the negative bands to the ionized molecule, and the lines to the atom; and that the molecule is ionized without dissociation.

PRINCETON UNIVERSITY.

36. THE RELATIVE INTENSITIES OF STARK EFFECT COMPONENTS IN THE HELIUM SPECTRUM.

BY JOHN STUART FOSTER.

A modified Lo Surdo tube was used with its axis perpendicular to the slit. The Stark effect components were *separate* and *parallel.*

The relative intensities of the p and of the s components of He 4922, 4472, and 4388 in an electric field of 15,000 v./cm. were measured by means of a neutral wedge. The *p* and *s* components in turn illuminated the slit of a large glass spectrograph. Energy distribution curves were calculated from the contour of the images and the corresponding wedge densities. The gas pressure was 2.8 mm.; the voltage on the tube 6000 v.; the current 3 mil. amps.; and the plates were exposed for 8 hours. The results for the main *p* components follow in order of increasing wave-length: for the line 4388, 8.2,[1] 4.1, 19.2, 28.0; for 4472, 168, 694; for 4922, 9.7, 31.6. Corresponding results for the *s* components are: for 4388, 1.0,[1] 3.0, 11.3, 9.5; for 4472, 76, 464; for 4922, 6.2, 21.4.

At present the method is limited in its usefulness by the appreciable pitting of the cathode, which occurs during long exposures. The investigation is being continued.

YALE UNIVERSITY.

37. CATHODOLYMINESCENCE OF HEATED ALUMINA.

BY D. T. WILBER.

IT is found that ordinary powdered alumina built up into a fritted cone in the oxyhydrogen burner, when freshly prepared, shows colored bands under cathode rays. The individual spectra causing these are the red ruby lines, a green, a blue and an orange band. The ruby bands are due to alumina with minute traces of chromium, for a clear white synthetic sapphire showed a brilliant red cathodoluminescence. A mixture of alumina and magnesia in molecular proportions with one per cent. of manganese gave a brilliant green band. Alumina and lime with manganese as active metal gave a broad band in the yellow and green. Alumina and lime with copper gave a band in the blue.

The layers of color on the cone seem to be due to the fact that the various colors come out at various temperatures and are destroyed at other higher temperatures. The green develops at the lowest temperature about 900° and disappears at about 1300° C. The ruby red appears soon after the green and persists through melting. The blue appears near the melting-point. These effects are much reduced if the material is completely fused.

CORNELL UNIVERSITY.

38. RADIATION FROM OXIDES. I. EMISSIVE POWER OF ZIRCONIUM OXIDE.

BY W. E. FORSYTHE.

THE emissive power of zirconium oxide for red radiation ($\lambda = .665\ \mu$) has been measured by two methods. The first consisted of heating on a platinum strip a small piece of the oxide, which had been baked at a temperature of about 1800° K. and measuring the relative brightness of the oxide and the

[1] "Isolated."

platinum strip. From these measurements both the temperature and emissive power can be calculated.

The second method consisted in pressing the oxide in the form of hollow glowers and then making a small radial hole, from which the true temperature could be determined. This is the method used by A. G. Worthing of this laboratory in determining the emissive power of tungsten.

The results showed a rather wide range with an average of about 16 per cent. which does not appear to vary greatly within the temperature range (1600° to 2600° K.).

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
April 5, 1922.

39. VARIATION OF CONTACT DIFFERENCE OF POTENTIAL WITH TEMPERATURE IN POTASSIUM AND SODIUM, AND THE ACCOMPANYING CHANGES IN PHOTO-ELECTRIC EMISSION.

BY HERBERT E. IVES.

IF a tungsten filament is mounted in the middle of a highly evacuated glass bulb, whose walls are covered with a layer of distilled potassium in contact with an electrode through the glass, it is possible, upon raising the filament to incandescence, to obtain a thermionic current between the filament and the potassium. The voltage-current relation is of the usual type. If now the bulb is immersed in liquid air, it is found that the thermionic voltage-current curve is laterally shifted by a fraction of a volt, indicating that the potassium has become electronegative compared with its room temperature condition.

According to present photoelectric theory a change in the contact difference of potential should be accompanied by a shift in the photoelectric threshold, which in turn may be expected to go along with a change in the total photo-electric emission. Such a shift in the long wave limit, and a corresponding change in the total emission, are actually found. The photoelectric current is diminished upon dropping to liquid air temperature to about 80 per cent. of its original value, for blue light, and to progressively lower values in the green and yellow. Most of the change occurs below zero Centigrade, which fact, together with the preponderance of sensitiveness in the blue region where the effect is least, probably explain the previous failures to observe a dependence of the photoelectric effect on temperature. The same phenomenon is found to a lesser degree in sodium.

RESEARCH LABORATORIES, AMERICAN TELEPHONE AND TELEGRAPH COMPANY AND WESTERN ELECTRIC COMPANY, NEW YORK,
March 31, 1922.

40. A Precision Rotating Sector: Aperture Variable and Measurable while in Motion.

By A. H. Pfund.

The outstanding defect in the well-known rotating sector whose aperture is variable while in motion lies in the mechanism for reading the angular opening. Due to the unavoidable "lost motion," the readings obtained from a scale over which a lever handle (whose motion is transmitted by clutches, gears, etc., to the sectors) moves, do not indicate the true angular openings of the sectored discs with precision.

This difficulty is overcome by means of a new stroboscopic device which makes possible the reading of graduated scale engraved on the rotating sectors. A narrow slit is cut into one of the two discs near the fiducial mark. The light from a small incandescent lamp passes through this slit and falls on a concave mirror which focuses a sharp slit image on the fiducial mark and on the adjacent portions of the graduated scale (engraved on the second disc). A central opening, fitted with a magnifying lens, is cut into the concave mirror for purposes of observation. The angular opening is varied by a mechanism resembling that employed by Abney and others. When the sectors are rotated rapidly, the scale appears to be absolutely at rest. Even minute imperfections in the scale are depicted with astonishing clearness.

A further novelty lies in the sectored discs themselves. They are so cut as to present two adjacent fields of view. As the angular opening is varied, one field increases in brightness while the other decreases. In extreme positions, one field is black while the other is at maximum brightness—conditions are reversed as the sectors rotate, relatively, through 90°.

Johns Hopkins University,
March, 1922.

41. Specific Intensity of Radiation in an Absorbing Medium.

By Leigh Page.

In 1882 Kirchhoff showed that the specific intensity of radiation in a transparent medium, multiplied by the square of the phase velocity, should be the same for all media in thermal equilibrium with one another. In his book on heat radiation Planck assumes this relation to hold for absorbing media. Laue has questioned this assumption and published an analysis of the problem (1910) which seems to show that the real part of the index of refraction should be replaced by the modulus.

In Laue's paper, it is assumed that a train of plane waves incident internally on the smooth surface of an absorbing medium, and the reflected train, can be treated as separated entities insofar as the propagation of energy is concerned. This, however, is not permissible, on account of the change in phase occurring on reflection. A more detailed investigation of the problem confirms Planck's assumption for moderately absorbing media. For strongly absorbing media

it is doubtful if the substitution of trains of plane waves for the irregular radiation due to absorption and subsequent emission is legitimate.

YALE UNIVERSITY.

42. PHENOMENA IN GASES EXCITED BY RADIO FREQUENCY CURRENTS.

BY E. O. HULBURT.

THE spectra from hydrogen, oxygen and air excited by currents of the frequency of the order of 10^6 alternations per second produced by an oscillating fifty-watt electron tube were photographed with a grating spectrograph in the spectral region from 5000 Å. to 3500 Å. These spectra were found to be identical with the spectra of the same gas stimulated by sixty-cycle current of the same strength as the radio frequency current.

The potentials just sufficient to set up luminosity in hydrogen and in oxygen were measured for direct and radio frequency, *i.e.*, frequencies from 10^5 to 10^7, current using electrode distances from 5 to 30 mm. and gas pressures from 1 to 5 mm. of mercury. For a specified gas pressure and electrode distance the direct potential and the maximum value of the radio frequency potentials were found to be the same. In the light of this result theoretical consideration indicated that the luminosity was started by collisions of electrons rather than of gaseous ions, with the gas molecules.

The flashes of light from hydrogen, oxygen, argon and air in turn excited by damped radio frequency current were found to be each a train of radio frequency flash s when examined by means of a rotating mirror. For hydrogen and argon the first few flashes of the radio frequency train showed a predominance of the green and blue lines of the spectrum, the later flashes of the train showed a predominance of the red lines. This was ascribed to the fact that the maximum value of the current in the first few cycles of the damped discharge was much greater than in the subsequent cycles.

STATE UNIVERSITY OF IOWA,
March, 1922.

43. ON THE EFFECT OF SHORT ELECTROMAGNETIC WAVES ON A BEAM OF ELECTRONS.

BY C. J. LAPP.

MUCH has been written concerning discontinuous wave fronts of electromagnetic radiation. Sir J. J. Thomson and A. Einstein have definitely developed such theories. As an experimental test apparatus[1] was devised by means of which electrons were shot through a compact beam of radiation such as ultraviolet light or x-rays. In order to magnify any possible effect the electron beam was twisted into a long spiral of about 3 cm. pitch and 1.5 cm. mean diameter by means of a strong magnetic field. The helical beam thus formed was about 70 cm. long, and the path of each individual electron much

[1] Knipp, C. T., PHYS. REV., Vol. 34, p. 477.

longer. If the electron beam appeared continuously and it was permitted to fall on a photographic plate, traces made on the plate when the radiation was turned off should be different from those made when the radiation was present if any effect occurred.

The experiment was performed, using as a source of electrons an especially prepared Wehnelt Cathode of microscopic size described in the "Transactions of the Illinois State Academy of Science," Vol. 14, 1921. Due to the velocity distribution of the electrons in the beams, the traces made on the photographic plates were almost circular. Photographs were taken, first, of the electron beam alone, and second, of the beam when intense radiation was directed across its path near the electron source.

A long series of exposures taken when ultraviolet light was used as the radiation, showed a slight scattering of the electrons which was but little larger than the experimental error.

When hard x-rays were used as radiation, a marked effect was found. The exposures taken when the electrons had passed through the x-rays showed a distinct scattering effect. This is undoubtedly evidence in favor of a discontinuous wave front theory.

UNIVERSITY OF ILLINOIS.

44. EFFECT OF THE INITIAL EMISSION VELOCITIES OF ELECTRONS ON THE MINIMUM ARCING VOLTAGE IN GASES.

BY K. T. COMPTON AND Y. T. YAO.

THE maintenance of arcs at considerably less than half the minimum ionizing potential [1] is a fatal objection to the theory of low voltage arcs as due to cumulative ionization[2] unless the difference between the theoretical and observed voltage can be accounted for as due to initial velocity of emission of electrons from the cathode.

Minimum Striking Voltage.—It is shown that the observed striking voltage should be equal to the difference between the minimum ionizing and radiating potentials, *less* the average energy of emission of electrons from the cathode, in equivalent volts.

Minimum Maintaining Voltage.—On the assumption that it requires the same number of electrons, with velocities exceeding the critical value, to maintain the arc as to cause it to strike, it is shown that the arc may be maintained at a voltage less than the striking voltage by an amount calculable from the relative increase of current when the arc strikes and the known distribution of velocities among the emitted electrons.

Comparison with Experiment.—Curves showing the relation between arc current and applied voltage at various vapor pressures and cathode temperatures in mercury vapor, agree with the above theory within the limits of experimental accuracy.

PRINCETON UNIVERSITY.

45. THE EFFECT OF LIQUID SURFACE UPON THE ARCING VOLTAGE IN MERCURY VAPOR.

BY Y. T. YAO.

LOW voltage arc in mercury vapor was studied under identical conditions for the two cases: (1) when the vapor was near the liquid surface, and (2) when it was far removed from it. It was found that there was an essential difference in the value of the striking voltage. In the first case, the 4.9-volt arc was prominent, whereas in the second case the 6.8-volt arc was apparently the only one present. It is suggested that the same entities which are active in fluorescence and resonance may also be responsible for the 4.9-volt arc in mercury vapor, since R. W. Wood has shown that fluorescence and resonance are excited in mercury vapor by the 2536 Å. line only when the vapor has been freshly distilled from the liquid surface.[1] Experiments along these lines are in progress.

PRINCETON UNIVERSITY.

46. MOBILITIES OF ELECTRONS IN HYDROGEN.

BY LEONARD B. LOEB.

IN a recent paper published in the PHYSICAL REVIEW[2] the writer gave the results of the measurement of the mobilities of electrons in nitrogen. This work has been extended to H_2 with a considerable improvement in the oscillating system and an increase in the accuracy of the measurements. This increased accuracy led to an elimination of certain "ageing" effects which had distorted the voltage-mobility curves in N_2. It led also to the explanation of the asymptotic feet of the mobility curves in H_2 and N_2 as being due to the diffusion of ions formed between the plates, to the electrometer plate. It therefore became possible to eliminate these feet completely. As in N_2 it was found that the mobility constant reduced to 760 mm. pressure varied as a function of the field strength and pressure. The mobilities obtained are very closely given by the equation,

$$K = \frac{2.74 \times 10^6}{399 + \frac{V}{d}\left(\frac{760}{p}\right)^{3/4}} \qquad (1)$$

or nearly as well by,

$$\left(K = \frac{1.905 \times 10^7}{81.3 + \frac{V}{d}\frac{760}{p}}\right)^{3/4}, \qquad (2)$$

where V is the potential difference of the alternating potential in volts, d is the distance between the plates, and p the pressure in mm. of Hg. The difference in the shape of the curves given by equations (1) and (2) is very slight.

[1] R. W. Wood, Astrophys. Jour., Vol. LIV., p. 149, 1921.

[2] L. B. Loeb, PHYS. REV., Vol. XIX., No. 1, pp. 224–38, Jan., 1922.

The fit of the curves is perhaps a trifle better in the case of equation (2), while (1) reproduces the family of curves more accurately. The form of the equation for mobility in H_2 differs from that observed in N_2 in that it has the power 3/4 instead of 1, as was originally found for N_2. Indications are that when the measurements are made in N_2 with the same precision as that achieved in H_2, the equation for the mobility in N_2 will be similar to the one in H_2. The new equation obtained in H_2 is more nearly of the form to be expected on theoretical grounds than the one found for N_2. It still however indicates that either the mean free path of the electron or the elasticity of the electron impact is a function of its velocity.

The values of K found for H_2 are in general much lower than the corresponding values found for N_2. If the values of K be taken for $V = 0$, *i.e.*, for electrons in thermal equilibrium with the gas molecules, the value of $K = 6860$ cm./sec. volt/cm. is obtained. The value of K computed from the theoretical equation for electron mobility in H_2, where $V = 0$, is $K = 13{,}060$ cm./sec. volt/cm. This indicates that the mean free path of electrons in H_2 is 0.53 of the value assumed by the kinetic theory of gases. Such a decrease in the mean free path of electrons could be caused by the attractive forces exerted by the electrons on the gas molecules. In the case of N_2 the value of K observed was 3.9 times the theoretical value. This indicates an abnormally great mean free path in N_2 which is at present unexplained. It is perhaps similar to the abnormal mean free path found in argon, independently by Ramsauer,[1] H. F. Mayer,[2] and Townsend.[3]

RYERSON PHYSICAL LABORATORY,
April 4, 1922.

47. THE ELECTRON EMISSION FROM THORIATED TUNGSTEN FILAMENTS.

BY IRVING LANGMUIR.

HEATING a tungsten filament containing thoria to 2800° K. causes a small fraction of the thoria to be reduced to metallic thorium distributed throughout the filament. At 2100° K. the evaporation of thorium becomes negligible and with exceptionally high vacuum the thorium, by diffusion, forms an adsorbed film covering a fraction θ of the surface with a layer of thorium atoms. The electron emission is measured at 1300° to 1800° where no diffusion occurs.

Theory and experiment show that the logarithm of the emission is a linear function of θ and the changes in θ caused by heat treatment are given by

$$N_0 \frac{d\theta}{dt} = DC_1(1 - \theta) - \nu,$$

where N_0 = Number of thorium atoms per cm.^{2} when $\theta = 1$; D = Diffusion coefficient (cm.^{2} sec.$^{-1}$) of thorium in tungsten; C_1 = Concentration gradient

[1] C. Ramsauer, Ann. d. Phys., 64, p. 513, 1921.
[2] H. F. Mayer, Ann. d. Phys., 64, p. 451, 1921.
[3] J. S. Townsend and V. A. Bailey, Phil. Mag., Vol. 43, No. 255, pp. 593–600, March, 1922.

at the surface (atoms of Th $\times$ cm.$^{-4}$); ν = rate of evaporation of thorium (atoms cm.$^{-2}$ sec.$^{-1}$) from a tungsten surface. Experiments show that

$$\log_{10} D = 0.21 - \frac{20900}{T},$$

$$\log_{10} \nu = 7.76 - \frac{44500}{T}.$$

Thorium atoms evaporate from underlying thorium atoms at rates very high compared to that of thorium from underlying tungsten atoms. Only for small values of θ (less than 0.1) is ν proportional to θ. For $\theta = 0.2$ to 0.3, ν reaches a maximum and then decreases slightly (about 20 per cent.) when $\theta = 0.9$.

RESEARCH LABORATORY, GENERAL ELECTRIC CO., SCHENECTADY, N. Y.

48. THE REMOVAL OF THORIUM FROM THE SURFACE OF A THORIATED TUNGSTEN FILAMENT BY BOMBARDMENT WITH POSITIVE IONS.

BY K. H. KINGDON AND IRVING LANGMUIR.

THE experimental tube contains a molybdenum cylinder *c* with closed ends, through which pass two thoriated tungsten filaments, *a* and *b*. The tube is filled to about 0.01 mm. pressure with the gas whose ions are to be used, and ions are formed by passing an electron current from *b* to *c*. The voltage between *b* and *c* is kept as low as possible, while *a* is kept at a relatively high negative potential to *b*, and can thus be bombarded with positive ions of any desired energy. The removal of Th from the surface of *a* by this bombardment can be studied from the change in the electron emission of *a*. He and Ne have been used.

Conclusions:—(1) Ne ions remove Th faster than He ions of the same energy.

(2) The amount of Th removed per second does not increase in proportion to the amount of Th on the surface, and is independent of the temperature.

(3) At 500 volts only about one He ion in 190 succeeds in removing a Th atom from the surface.

(4) The rate of removal of Th increases approximately linearly with the energy of the ions; and is proportional to the positive ion current.

RESEARCH LABORATORY, GENERAL ELECTRIC CO., SCHENECTADY, N. Y.

49. THE MEASUREMENT OF MAGNETIC FIELD STRENGTH BY MEANS OF ELECTRON TUBES.

BY ALBERT W. HULL.

THE following method of calibrating magnetic field coils has been in use in the Research Laboratory for the past two years:

A two-electrode tube of circular cylindrical symmetry is connected in multiple with a standard voltmeter, and in series with a high resistance and a source of voltage.

The electron tube is placed in the field and rotated until the voltmeter reading is a maximum. The direction of the field is then given by the direction of the axis of the tube, and its magnitude H by the voltmeter reading V, according to the equation

$$H = \frac{6.72}{R}\sqrt{V},$$

where R is the radius of the anode.

The readings are independent of both filament temperature and generator voltage within wide limits, so that any available power supply may be used; and can be relied upon, *without calibration*, to within 5 per cent. in absolute value, and to within ¼ per cent. for relative values over a short range. Fields from 20 to 500 gauss can be measured in this way by appropriate choice of resistance and voltage.

An alternative method is to connect an electron tube of any type in *series* with a voltmeter and source of voltage. Calibration is necessary. The range is from 10 to 500 gauss. By using a symmetrical tube and a polarizing field coil around on the tube, the range may be extended down to 1/10 gauss.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.,
November 22, 1921.

50. A GENERAL RELATION FOR ELECTRON EMISSION FROM METALS.

BY SAUL DUSHMAN.

ON the basis of the Nernst Heat Theorem it has been shown that the vapor pressure of a monatomic substance is given by a relation of the form

$$ln\, p = -\frac{L_0}{RT} + \frac{5}{2} ln\, T + C. \qquad (1)$$

According to the quantum theory, C, the so-called chemical constant, may be calculated from the equation

$$C = ln\left(\frac{lu\ (2\pi)^{3/2}k^{5/2}M^{3/2}}{N^{3/2}h^3}.\right) \qquad (2)$$

Tolman has applied equation (1) to electron emission, by using the value of C derived from the vapor pressure data for helium. Using equation (2) to calculate C, and applying equation (1) to electron emission, it can be shown that the *emission from any substance*, in amperes per cm.^{2}, is given by the relation

$$i = 60.2T^2\epsilon^{-b_0/T} \qquad (3)$$

where $60.2 = \frac{10}{C}\left(\frac{2\pi k^2 me}{h^3}\right)$

This equation is found to be in very satisfactory agreement with electron emission data obtained from pure tungsten, and unimolecular layers of thorium

and calcium on tungsten. The following values of b_0 have been obtained for these metals.

W	51,500 ± 500,
Th	34,000 ± 500,
Ca	26,000 ± 1,000.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

51. THE SECONDARY ELECTRON EMISSION FROM NICKEL.

BY C. DAVISSON AND C. H. KUNSMAN.

THE total number of secondary electrons emitted per primary bombarding electron has been measured for nickel for bombarding potentials V, up to 350 volts, by the same method employed by Millikan and Barber (Proc. Nat. Acad., Jan., 1921) for copper.

The emission factor has a value of 1.08 for $V = 350$ volts, and decreases to 0.32 for $V = 15$ volts, and then remains practically constant at 0.30 for all speeds to $V = 0$. While the bulk of the secondary emission from nickel is made up of slow-speed electrons, electrons of all speeds up to the speed of the primaries are to be found at all bombarding potentials. The presence of high-speed electrons have not in general been observed in the investigations on the emission from metals.

Apparently the emission at the lower voltages is due to the escape from the metal of primary electrons that may on the average have had a considerable number of elastic encounters with atoms in and near the surface of the target. At high voltages the high-speed electrons escape from the target as the result of single encounters between primary electrons and atoms.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH CO.,
AND THE WESTERN ELECTRIC COMPANY, INC.,
April 19, 1922.

52. AN EXPERIMENTAL METHOD OF TESTING THE EXPRESSIONS FOR LONGITUDINAL AND TRANSVERSE MASSES OF THE ELECTRON.

BY L. T. JONES AND W. C. POMEROY.

EXPRESSIONS for the longitudinal and transverse masses of the electron have been deduced by Lorentz-Einstein, Abraham, Bucherer-Langevin and Walker. An experimental method of testing these expressions is proposed making use of an electron stream from a hot filament cathode, the electrons being accelerated by the sinusoidal voltage from a high potential transformer; thus in the beam studied a large range of electron velocities is present. Each electron, upon acquiring its maximum velocity, enters an electrostatic field, between parallel horizontal plates, which is a constant fraction of and at all times in phase with the variable accelerating field. Coincident with this field

is a constant magnetic field. These fields produce downward and sidewise deflections, respectively, which depend in magnitude upon the velocity of the electron. A curve is traced upon a photographic plate placed on the lower of the horizontal plates. This may be compared with the theoretical curves, which have been calculated for each of the four theories. A plot of the calculated curves indicates the possibility of confirmation of one of the theories. The experimental work is proceeding.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA.

53. OIL-DROP EXPERIMENTS AS PROOF OF THE INVARIANCE OF ELECTRIC CHARGE.

BY F. W. LOOMIS.

AT the last meeting I pointed out [1] that the fact that all of Millikan's oil drops could be made electrically neutral within 1/1000 of an electronic charge showed that the charge on an electron is equal to that on a proton to within 1 part in 4×10^{17}. This is true for the electrons composing substances as diverse as mercury and oil. It follows that all electrons have, to this order of accuracy, the same charge; and since the electrons in different substances have widely different velocities, this serves as a confirmation of the relativity law of the invariance of electric charge. A rough calculation can be made of the limit set by these experiments on the possible variation of charge with velocity.

NEW YORK UNIVERSITY.

54. STUDY OF THE HYDROGEN RADIATION IN THE RED AND THE INFRA-RED.

BY F. S. BRACKETT.

Hydrogen lines have been observed at wave-lengths $\lambda = 4.0^5\,\mu$ and $2.6^3\,\mu$ due, according to Bohr's theory, to an electron falling into the fourth from the fifth and sixth rings respectively—forming the *first two members* of a *new series*.

Lines have also been observed at wave-lengths $\lambda = 1.8^8\,\mu$, $1.2^8\,\mu$, $1.0^9\,\mu$, $1.0^1\,\mu$ and $0.95\,\mu$. These form the *first five members of the Paschen series*, due to an electron falling into the third from the fourth, fifth, sixth, seventh and eighth rings respectively. The first two were observed and accurately measured by Paschen.

The relative intensities of H_α and the first Paschen line are found to vary with the current. In some cases the first Paschen line was found to be more intense than H_α in the ratio 4 : 3. The first line of the new series was found to be about one fourth the intensity of H_α.

The *variation* of *intensity* of Hydrogen lines is *not a continuous function of current intensity*. *Discontinuities* occur for currents corresponding to the *threshold* (resonance) *potentials* predicted by Sommerfeld. Those have been

[1] Paper 15, 113th meeting.

identified which *correspond to* amounts of energy necessary in order to expel an electron from the *first* to the *third, fourth, fifth, sixth, seventh* and *eighth rings* respectively, also probably the ninth, tenth, eleventh and twelfth rings. Other discontinuities have also been observed which would not be expected from theory.

JOHNS HOPKINS UNIVERSITY,
April 7, 1922.

55. THE VOLTAGE-CURRENT RELATION IN CENTRAL ANODE PHOTO-ELECTRIC CELLS.

BY HERBERT E. IVES AND THORNTON C. FRY.

IT is found experimentally that the voltage photo-electric current relation in central anode photo-electric cells is different for different colors. On the assumption that the electrons are emitted from the walls in all directions according to a cosine distribution, and with all velocities up to the limit set by the quantum relation, according to a parabolic distribution law of energies, a mathematical theory is developed. The electrons describe elliptic orbits, determined by their initial direction and velocity, and the electric field. It is shown that very small initial velocity differences are only overcome by large electric fields. For an assumed case of a threshold at 6000 Å.U. saturation is calculated to occur at 40 volts for emission due to 5500 Å.U. radiation, and at 160 volts for emission due to 4500 Å.U. radiation. These differences agree as to order of magnitude with the experimentally observed ones.

RESEARCH LABORATORIES, AMERICAN TELEPHONE
AND TELEGRAPH COMPANY AND WESTERN ELECTRIC
COMPANY, NEW YORK,
March 31, 1922.

56. THE BROADENING OF THE BALMER LINES OF HYDROGEN WITH PRESSURE.

BY E. O. HULBURT.

SPECTROGRAMS were taken of H_β, H_γ and H_δ of hydrogen excited by condensed discharges for gas pressures from 2 to 135 mm. of mercury. At the highest pressure the photographic widths of the lines were nearly 100 Å. Throughout the series of pressures the intensity of the central portion of H_β was maintained sensibly constant. Assuming that the widening was caused by disturbance of the uniformity of the radiation due to collisions of the radiating particles (as proposed by others), the widening of the lines with pressure was calculated and found to be considerably less than that observed. This result indicated that the assumption was an improper one, and that the widening should be attributed to other causes, such as the effect of the electrical fields, as investigated by Merton (Proc. Roy. Soc., 92, 322, 1915).

STATE UNIVERSITY OF IOWA,
April, 1922.

57. CRYSTAL STRUCTURES OF VANADIUM, GERMANIUM AND GRAPHITE.

BY ALBERT W. HULL.

VANADIUM has been analyzed in powdered form by the x-ray diffraction photograph method. Its lattice is a body-centered cube with side 3.04 Å. The distance between nearest atoms is 2.63 Å, and the ideal density 5.96.

Germanium is found to have a crystal structure the same as that of diamond. The side of the unit cube is 5.63 Å, the distance between the nearest atoms 1.218 Å, and the density 5.36.

New and more accurate diffraction patterns of graphite have confirmed the structure described in an earlier paper, viz., a lattice of the hexagonal close-packed type. The measurements can not be reconciled with the rhombohedral structure found by Debye. The exact distance between nearest planes can not yet be stated with certainty. Measurements are in progress which, it is hoped, will determine this distance.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

58. THE DIFFUSION COEFFICIENT IN SOLIDS AND ITS TEMPERATURE COEFFICIENT.

BY SAUL DUSHMAN AND IRVING LANGMUIR.

CONSIDER a solid as consisting of layers of atoms, δ being the distance between layers. Let Kdt be the probability that a given atom in a layer A shall move into the adjacent layer B during the time dt. Then the diffusion coefficient is

$$D = K\delta^2. \tag{1}$$

The quantity K is a frequency analogous to the velocity constant of a unimolecular chemical reaction. The velocity of such reactions is approximately given by the semi-empirical relation

$$K = \nu e^{-h\nu/kT} \quad \text{or} \quad K = \frac{Q}{Nh} e^{-Q/RT}, \tag{2}$$

where ν or Q is characteristic of the reaction. It is now shown that this relation also holds for the K calculated by (1) from the diffusion coefficient. For the diffusion of thorium through tungsten at 2300° K. $D = 1.1 \times 10^{-9}$ cm.^{2} sec.$^{-1}$. Substituting this in (1) and (2) gives $Q_{cal.} = 90{,}500$ g. calories per g. molecule. Plotting log $D_{obs.}$ against the reciprocal of the temperature, from 1900 to 2500° K., gives a straight line from whose slope we get $Q_{obs.} = 94{,}000 \pm 3{,}000$, in satisfactory agreement with $Q_{cal.}$. Even better agreement is obtained from published data of Roberts-Austin on diffusion of gold in lead and of Warburg and Schulze on diffusion of sodium ions in glass. Thus the temperature coefficient of D may be calculated if the value of D is known at one temperature.

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59. NOTE ON THE COHN-MINKOWSKI FORMULA FOR THE MOTION OF INSULATORS IN A MAGNETIC FIELD.

BY S. J. BARNETT.

ON the theory of Lorentz and Larmor the polarization produced by the motional electric intensity acting on an insulator with dielectric constant K and permeability unity moving with velocity v in a magnetic field with induction B is

$$P_1 = \frac{(K-1)}{4\pi}\frac{1}{c}[vB].$$

If the permeability is μ and the intensity of magnetization

$$I = \frac{B}{4\pi}\left(1 - \frac{1}{\mu}\right)$$

there is in addition an electric polarization

$$P_2 = \frac{1}{c}[vI] = \frac{1}{4\pi c}\left(1 - \frac{1}{\mu}\right)[vB]$$

due to the magnetization. This follows from a simple theorem of Maxwell's: The motion with velocity v of a constant electromagnetic system with vector potential A produces an electric field the potential of whose polar part is $\varphi = (Av)/c$. An immediate consequence of this is that the electric moment of the distribution produced by the motion is equal to $1/c[vm]$, where m is the magnetic moment. Applying this to the unit volume, we obtain the above expression for P_2. The total polarization is

$$P = P_1 + P_2 = \frac{K}{4\pi c}\left(1 - \frac{1}{\mu K}\right)[vB],$$

a result hitherto obtained only on the theories of Cohn and Minkowski. The result, like Maxwell's theorem, is correct only to quantities of the first order in v/c. Reference is made to results obtained by Budde, Lorentz, and others, which are implicit in Maxwell's theorem, the approximate character of which is due to its being based on a principle of relativity assuming equality of electromotive intensity and magnetic induction to fixed and moving observers.

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60. A NEW METHOD FOR THE GENERATION OF SOUND WAVES.

BY JULIUS HARTMANN. INTRODUCED BY HUGO FRICKE.

IN an air-jet, produced by a pressure higher than 1.9 Atm., there is a peculiar, periodic distribution of the pressure along its length, a demonstration of which can be made photographically by means of a special optical arrangement. If the aperture of a small cylindrical acoustic resonator is introduced into the

parts of the jet where the pressure is rising, a note is produced. The wave-length of this has been determined by a Kundt tube and is found to be determined by the equation

$$\lambda = 4l + 2.4d,$$

where l is the length and d the diameter of the resonator.

Notes of a frequency higher than 100,000 have been produced by means of this sound generator. The efficiency of the generator is about 10 per cent.

If the narrow aperture of a bulb is introduced in the jet, a note is also produced. The frequency of this note is mainly determined by the volume of the bulb, by the width of the aperture, and by the place in the jet where it is introduced. This method affords a very efficient siren.

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61. THE EFFECT OF A MAGNETIC FIELD ON THE ABSORPTION OF X-RAYS.

BY JOSEPH A. BECKER. INTRODUCED BY F. K. RICHTMYER.

Effect of a magnetic field on the absorption of x-rays, if it can be measured, would have an important bearing on the theory of atomic structure. Since the effect is very small, a differential method was adopted and by eliminating various spurious effects the apparatus was made sensitive enough to detect a change of 1 part in 10,000. Observations were made with a peak voltage of 80 across the Coolidge tube (mean wave-length about 0.3 Å.) and with a magnetic field H of about 18,000 gauss. For H perpendicular to the rays, *aluminum, carbon, copper, iron, nickel, platinum, zinc, and silver* showed changes in absorption coefficients of $+ 8 \pm 6$, $- 5.6 \pm 2$, $+ 0.8 \pm 0.4$, $- 10 \pm 2$, $+ 1.6 \pm 0.5$, $+ 1.7 \pm 0.4$, $- 1.2 \pm 0.4$ and $+ 1.6 \pm 0.8$, parts in 10,000 respectively, while with H parallel to the rays the corresponding changes were $+ 2.7 \pm 1$, $+ 3 \pm 1$, $+ 1.4 \pm 1$, $- 0.5 \pm 2$, $+ 0.7 \pm 2$, $+ 1.1 \pm 1$ and $+ 1.3 \pm 1 \times 10^{-4}$, silver not being tried. These results are in accord with the hypothesis that the magnetic properties are largely determined by the outer shell or valency electrons, since at the wave-lengths used by far the greater part of the absorption is due to the inner electrons. Wood was also tested with softer rays having a mean wave-length of about 1.2 Å., and showed a change of $+ 80 \pm 20 \times 10^{-4}$ as compared with 3×10^{-4} for carbon for wave-length 0.3 Å. Following the suggestion of this result it is proposed to do further work with light elements and softer x-rays.

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62. FORMATION AND LIFE OF METASTABLE HELIUM.

BY FABIAN M. KANNENSTINE.

HELIUM arcs were formed between a Wehnelt cathode and nickel anode in a pyrex tube containing carefully purified helium. A voltage of about 35 volts from a battery was impressed between the anode and cathode. The electrical

circuit was arranged so that the impressed voltage could be suddenly reduced. The arc current was passed through the deflecting coils of a sensitive Braun tube and the spot was examined directly and with a rotating mirror.

On suddenly reducing the voltage to a value below about 4 volts, it was seen in the rotating mirror that the arc was extinguished in a time too short to be so observed. When the lower voltage was increased to about 4 volts a sudden change occurred; the arc died out gradually in a time that could be readily observed, indicating the possibility of maintaining an arc in the metastable helium formed, with about 4 volts as long as any was present. With the lower voltage 22.6 volts the arc continued at a certain value for a brief time before dying out, while with 23.6 volts the arc current decreased in two stages, which is possibly connected with the fact of the existence of two radiating potentials in helium.

This interpretation in terms of the presence of metastable helium was also supported by Braun-tube oscillograms when the arc was excited by alternating electromotive forces of various frequencies. With 60 cycles the arc did not strike below the ionization potential of helium indicating that the metastable helium disappeared during the negative half of the wave. With 350 cycles the arc struck at about 4 volts showing that the metastable helium persisted during half a cycle.

The theoretical ionization potential of metastable helium is 4.8 volts and probably is to be identified with the value observed.

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